Institute and Faculty of Actuaries

On contemporary mortality models for actuarial use I: practice

Stephen J. Richards and Angus S. Macdonald

11th March 2025

Overview

Institute and Faculty of Actuaries

1. Improved data-quality checking
2. A better match to reality
3. Modelling rapid changes in risk
4. Better management information
5. Conclusions
6. Acknowledgements

1 Improved data-quality checking

Institute and Faculty of Actuaries

Kaplan-Meier

Institute and Faculty of Actuaries

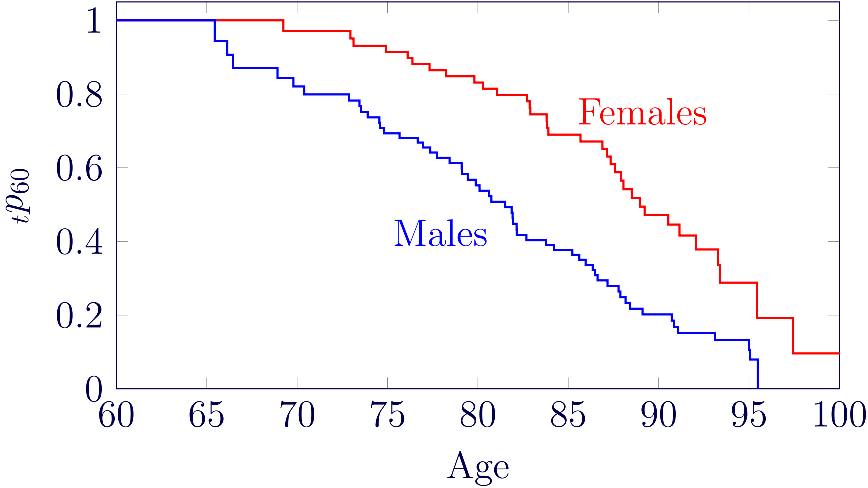
Kaplan and Meier [1958] presented a non-parametric estimate of the survival curve, ${}_tp_x$:

$$ {}_t\hat{p}_x = \prod_{t_i \le t} \left(1 - \frac{d_{x+t_i}}{l_{x+t_i^-}} \right), \qquad (1) $$

- x is the outset age for the survival function,
- $\{x + t_i\}$ is the set of distinct ages at death,
- $l_{x+t_i^-}$ is the number of lives alive immediately before age $x + t_i$ and
- d_{x+t_i} is the number of deaths occurring at age $x + t_i$.

Kaplan-Meier is a step function

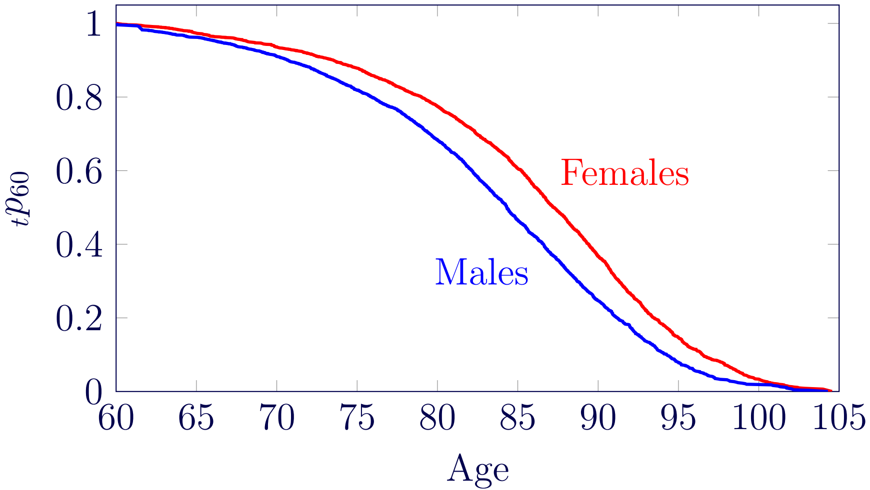
Mortality survival curves for home-reversion plans.



Source: Richards and Macdonald [2024, Figure 19].

Benefit 1: Data quality checks

Survival curves for Dutch pension scheme:

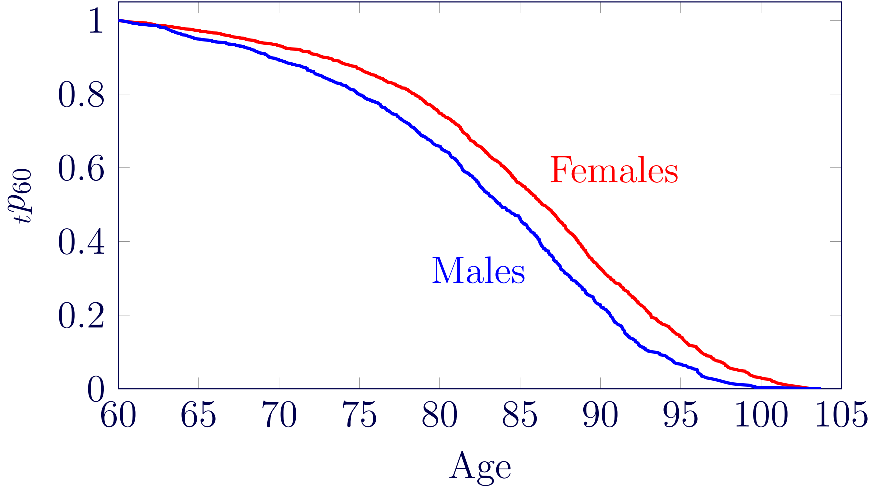


Source: past consulting work.

Benefit 1: Data quality checks

Institute and Faculty of Actuaries

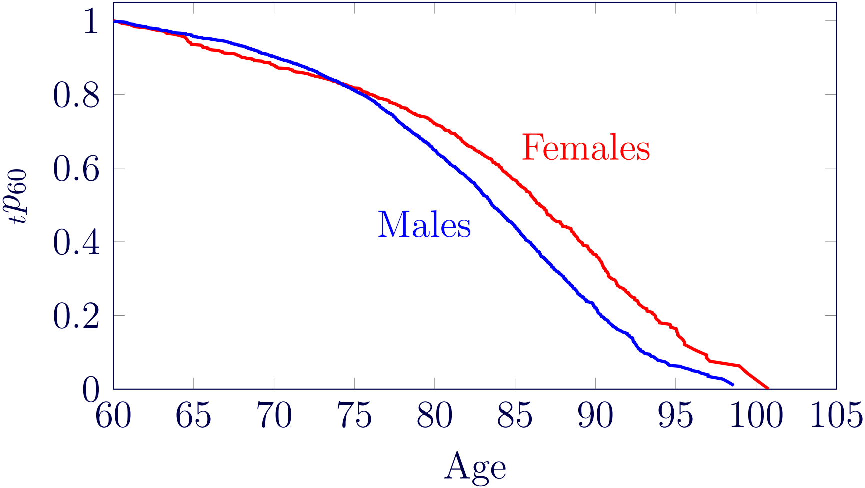
Survival curves for Scottish pension scheme:



Source: Richards and Macdonald [2024, Figure 12(a)].

Benefit 1: Data quality checks

Survival curves for UK pension scheme seeking longevity swap:



Source: current consulting work.

Kaplan-Meier estimator

Institute and Faculty of Actuaries

- Quickly becomes smooth for even small portfolios.
- Useful for communicating with non-specialists.
- Very useful data-quality check.

Question

- If this is so useful for actuarial work, why didn't actuaries invent it?
- One did — Böhmer [1912] at International Congress of Actuaries.
- Besides his academic actuarial work, Böhmer also worked for the German insurance regulator†.

†DGVFM [1957, page 134].

Böhmer and Kaplan-Meier

Böhmer [1912, equation 4]:

$$1-\gamma_h = \Pi_h \frac{A_n}{A_{n-1}}$$

Kaplan and Meier [1958, equation 2b]:

$$\hat{P}(t) = \prod_{j=1}^{k} (n_j'/n_j)$$

An explicitly actuarial method

Institute and Faculty of Actuaries

Böhmer [1912, page 331]:

	E_1	E_3	$\bar{E}$	E_1	E_2	$\bar{E}$	E_2	$\bar{E}$	E_1	E_1
n	1	2	3	4	5	6	7	8	9	10
A_n	284	283	284	283	282	283	282	283	282	281
A_{n-1}	285	284	283	284	283	282	283	282	283	282

- Event E_1 is death, the others being disability claim (E_2), voluntary withdrawal (E_3) and new entrant ($\bar{E}$).
- A_{n-1} is the number of lives immediately before an event, A_n is the number afterwards.
- $\hat{p}_{48} = \frac{284}{285} \cdot \frac{283}{284} \cdot \frac{282}{283} \cdot \frac{281}{282} = 0.985965$, so $\hat{q}_{48} = 0.0014035$.

2 A better match to reality

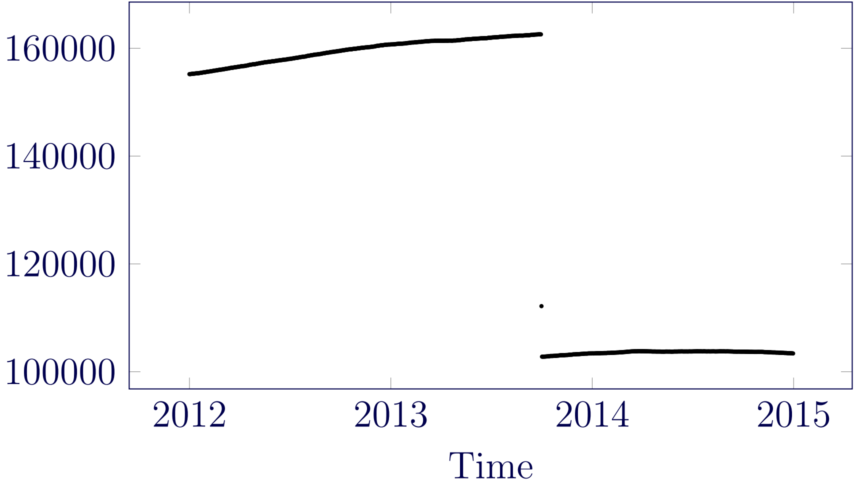
Benefit 2: Matching reality

Institute and Faculty of Actuaries

- A binomial mortality model is like a coin toss.
- A binomial trial must produce one of the two events allowed: death or survival.
- However, observation can be interrupted in real world...

Bulk transfers out

Number of in-force annuities at each date for a UK insurer:



Source: Richards and Macdonald [2024, Figure 3(a)].

Observation interruptus

Institute and Faculty of Actuaries

Observation can be interrupted mid-year by:

- Legal transfer of liabilities,
- Transfer to new administrator,
- Migration to a new administration system, or
- Commutation of small pensions.

Survival models

Institute and Faculty of Actuaries

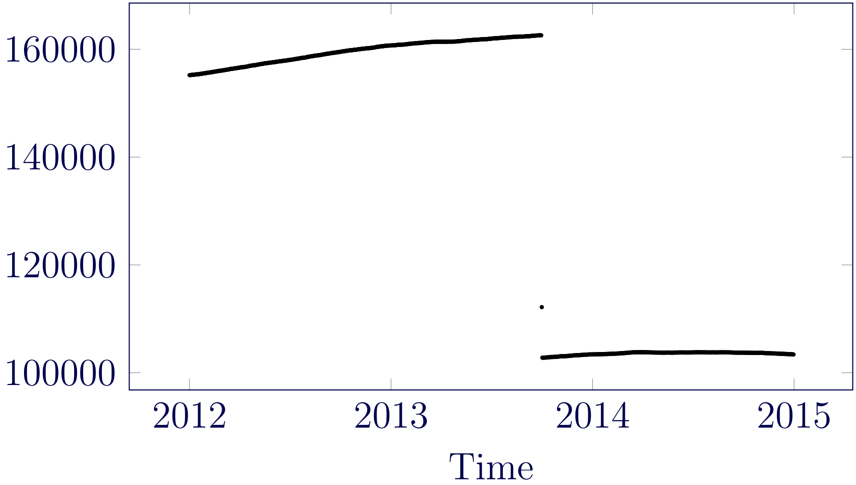
- Survival models handle interrupted observations as *right-censored* records.
- Early exits are treated like survivors, just with an earlier censoring date.

Mid-year additions

- A binomial mortality model assumes all lives are known at the start of the year.
- No facility for mid-year additions.
- However, new entrants during the year are routine. . .

Continuous new business

Number of in-force annuities at each date for a UK insurer:



Source: Richards and Macdonald [2024, Figure 3(a)].

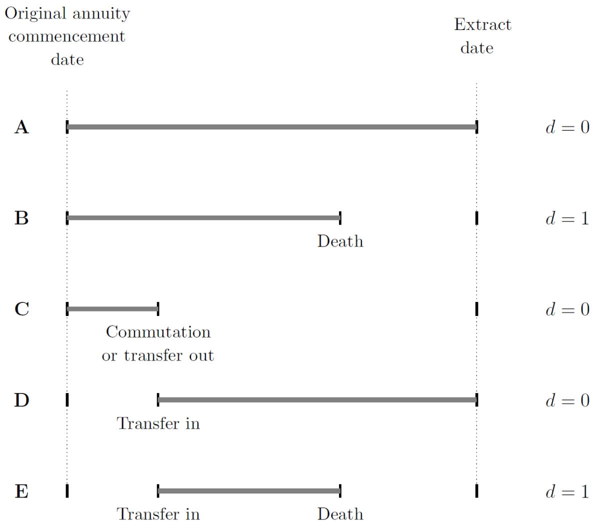
Benefit 2: Matching reality

Institute and Faculty of Actuaries

- Pension schemes and annuity portfolios are like medical trials:
  - Continuous recruitment (new retirals, surviving spouses).
  - Withdrawals/loss to follow-up (transfers out, commutation).
- Binomial models are not well suited to this...

...but survival models are.

Censoring and left-truncation



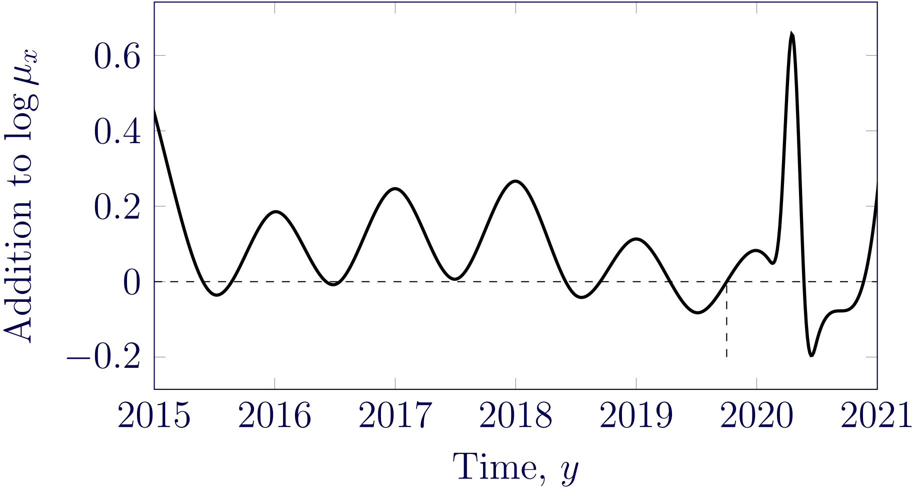
3 Modelling rapid changes in risk

Benefit 3: Rapid changes

Continuous-time modelling gives far greater insight into rapid changes.

Mortality levels over time

Period effects after allowing for age, sex and pension size:



Source: Richards [2022b, Figure 17(a)].

4 Better management information

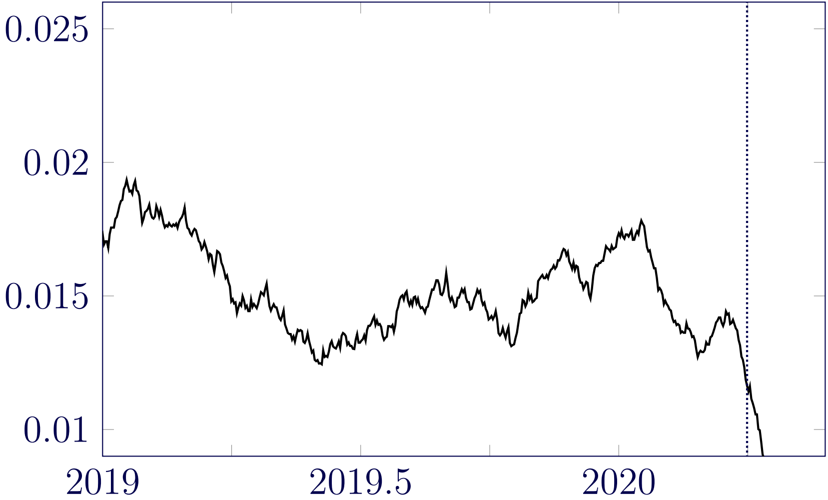
Institute and Faculty of Actuaries

Benefit 4: Management information

Management information

Institute and Faculty of Actuaries

Mortality hazard using June 2020 extract:



Source: Richards and Macdonald [2024, Figure 15(a)].

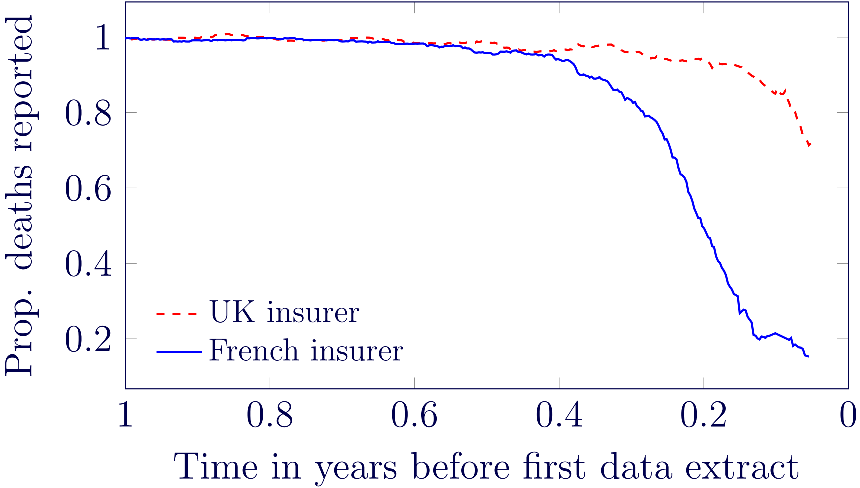
Management information

Institute and Faculty of Actuaries

1. No sign of pandemic mortality by June 2020.
2. Problem of delays in reporting deaths (IBNR/OBNR)...

Management information

Estimated proportion of deaths reported for two annuity portfolios:



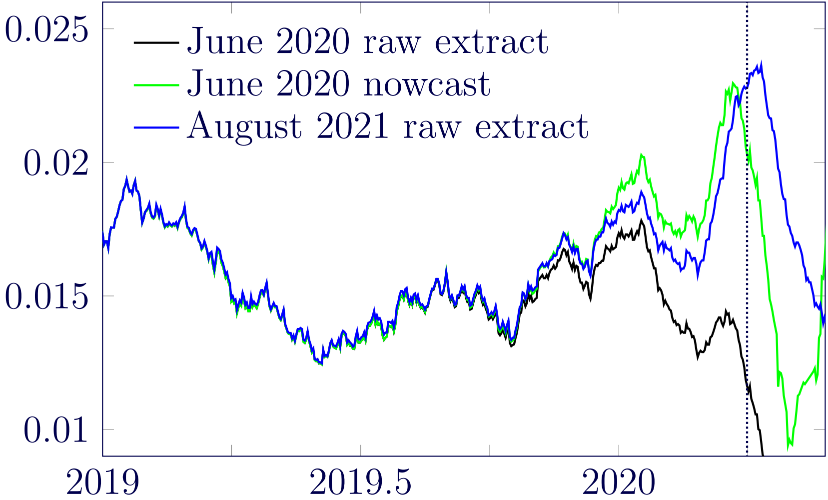
Source: Richards [2022a, Section 4].

Management information

1. Estimate the delay function.
2. Use this to "gross up" estimate of current mortality.
3. Bańbura et al. [2013] call this a "nowcast"...

Management information

Mortality hazard:



Source: Richards and Macdonald [2024, Figure 15].

5 Conclusions

5 Conclusions

Institute and Faculty of Actuaries

With continuous-time methods actuaries can:

1. Improve data-quality checking,
2. Match the reality of business processes,
3. Model rapid changes in risk, and
4. Get timelier management information.

6 Acknowledgements

Institute and Faculty of Actuaries

6 Acknowledgments

Thanks to:

- David Raymont of the IFoA for sourcing the original German-language text for Böhmer [1912] and
- Kai Kaufhold of AdRes for sourcing DGVFM [1957].

References I

M. Bańbura, D. Giannone, M. Modugno, and L. Reichlin. Chapter 4 — Now-Casting and the Real-Time Data Flow. In Graham Elliott and Allan Timmermann, editors, *Handbook of Economic Forecasting*, volume 2, pages 195–237. Elsevier, 2013. doi: 10.1016/B978-0-444-53683-9.00004-9.

P. E. Böhmer. Theorie der unabhängigen Wahrscheinlichkeiten. *Presented to the 7th International Congress of Actuaries, Amsterdam, 2-7 September, 1912*, pages 327–343, 1912.

References II

DGVFM. Paul Eugen Böhmer 80 Jahre. *Blätter der DGVFM*, 3(April 1957):133–134, 1957. doi: 10.1007/BF02808860.

E. L. Kaplan and P. Meier. Nonparametric estimation from incomplete observations. *Journal of the American Statistical Association*, 53:457–481, 1958.

S. J. Richards. Real-time measurement of portfolio mortality levels in the presence of shocks and reporting delays. *Annals of Actuarial Science*, 16(3): 430–452, 2022a. doi: 10.1017/S174849952200021.

References III

S. J. Richards. Allowing for shocks in portfolio mortality models. *British Actuarial Journal*, 27:1–22 (with discussion), 2022b. doi: 10.1017/S1357321721000180.

S. J. Richards and A. S. Macdonald. On Contemporary Mortality Models for Actuarial Use I - Practice. *Presented to the Institute and Faculty of Actuaries on 24th October 2024*, 2024.

7 Legal matters

Longevitas is a registered trademark:

- in the UK (No. 2434941),
- in the USA (No. 3707314), and
- in the European Union (No. 5854518).

LONGEVITAS™