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Some practical benefits of continuous-time methods

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2. Benefits of continuous time
3. Conclusions

**ON
CONTEMPORARY
MORTALITY
MODELS**
FOR ACTUARIAL USE

By S. J. Richards & A.S. Macdonald

The Longevity logo, consisting of a gear icon and the word "LONGEVITAS" in a serif font, positioned between two horizontal lines.

PDF available at: <https://www.longevity.co.uk/published-paper/contemporary-mortality-models-actuarial-use>

2 Benefits of continuous time

With continuous-time methods actuaries get:

1. Improved data-quality checking.
2. A better match to reality.
3. Modelling of rapid changes in risk.
4. Superior management information.

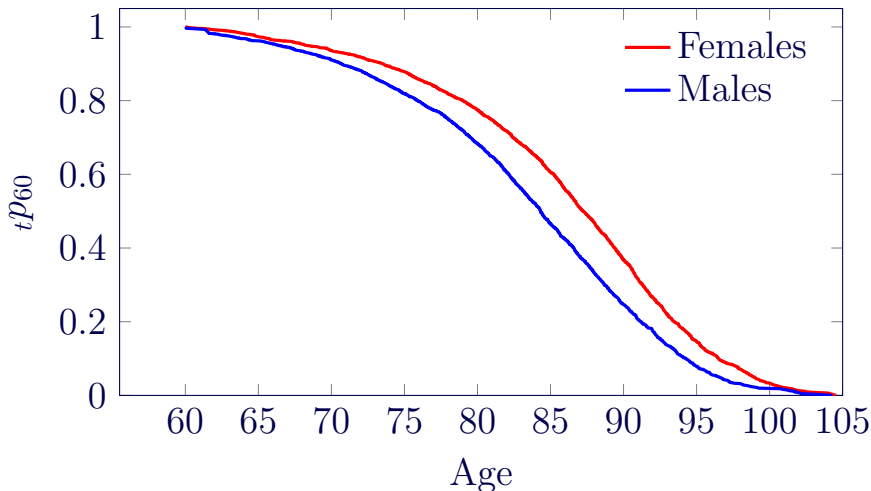
Kaplan and Meier [1958] presented a non-parametric estimate of the survival curve, ${}_t p_x$:

$${}_t \hat{p}_x = \prod_{t_i \leq t} \left(1 - \frac{d_{x+t_i}}{l_{x+t_i^-}} \right), \quad (1)$$

- x is the outset age for the survival function,
- $\{x + t_i\}$ is the set of distinct ages at death,
- $l_{x+t_i^-}$ is the number of lives alive immediately before age $x + t_i$ and
- d_{x+t_i} is the number of deaths occurring at age $x + t_i$.

Benefit 1: Data quality checks

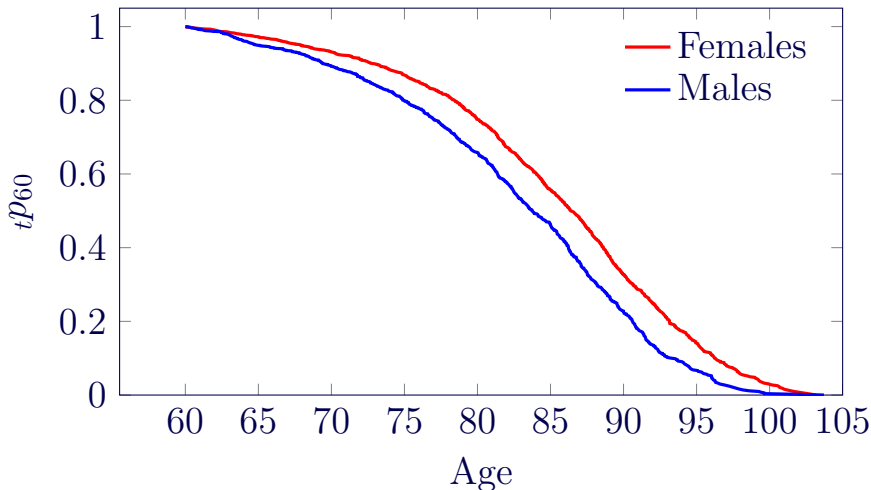
Survival curves for Dutch pension scheme:



Source: past consulting work.

Benefit 1: Data quality checks

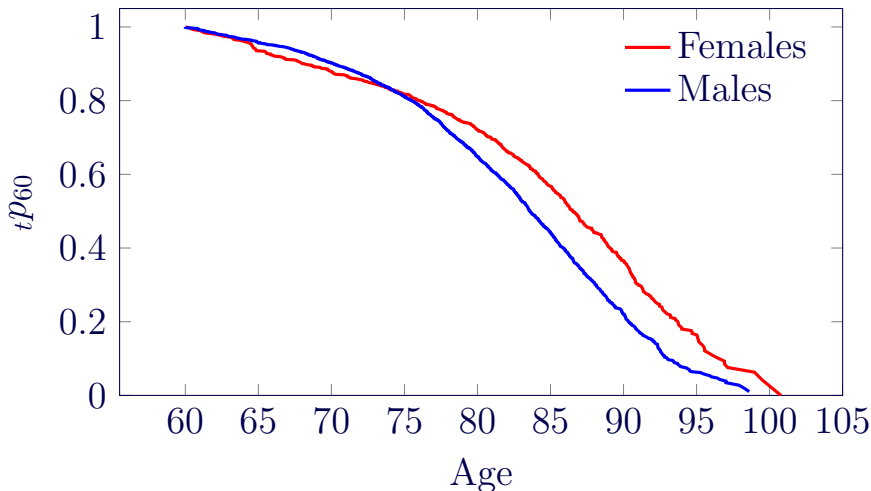
Survival curves for Scottish pension scheme:



Source: Richards and Macdonald [2024, Figure 12(a)].

Benefit 1: Data quality checks

Survival curves for UK pension scheme seeking longevity swap:



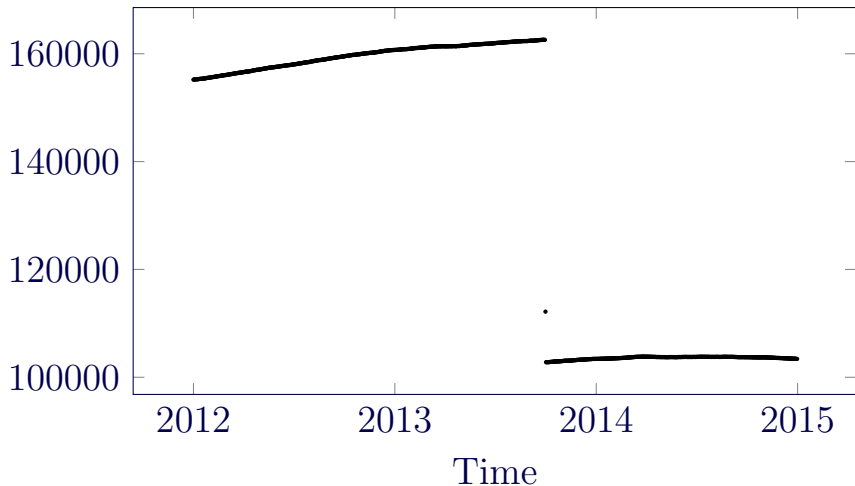
Source: current consulting work.

Kaplan-Meier estimates are useful:

- As checks for data quality.
- For communicating to non-specialists.

- A binomial mortality model is like a coin toss.
- A binomial trial must produce one of the two events allowed: death or survival.
- However, observation can be interrupted in real world...

Number of in-force annuities at each date for a UK insurer:



Source: Richards and Macdonald [2024, Figure 3(a)].

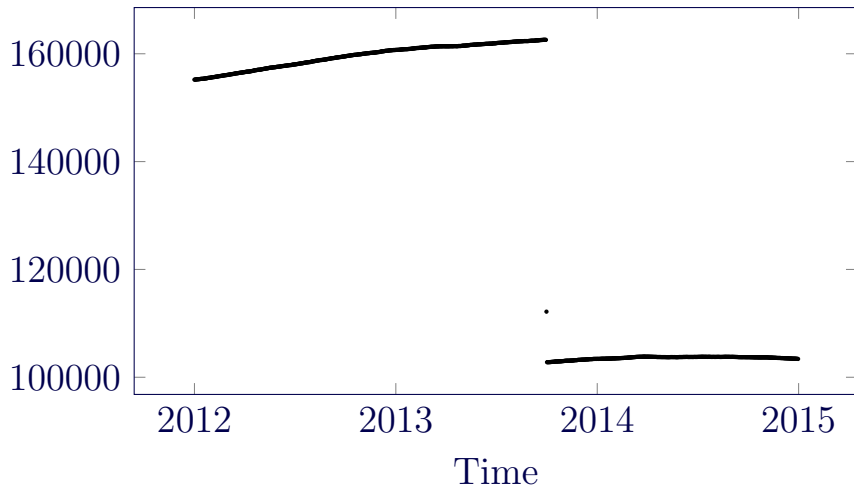
Observation can be interrupted mid-year by:

- Legal transfer of liabilities,
- Transfer to new administrator,
- Migration to a new administration system, or
- Commutation of small pensions.

- Survival models handle interrupted observations as *right-censored* records.
- Early exits are treated like survivors, just with an earlier censoring date.

- A binomial mortality model assumes all lives are known at the start of the year.
- No facility for mid-year additions.
- However, new entrants during the year are routine...

Number of in-force annuities at each date for a UK insurer:

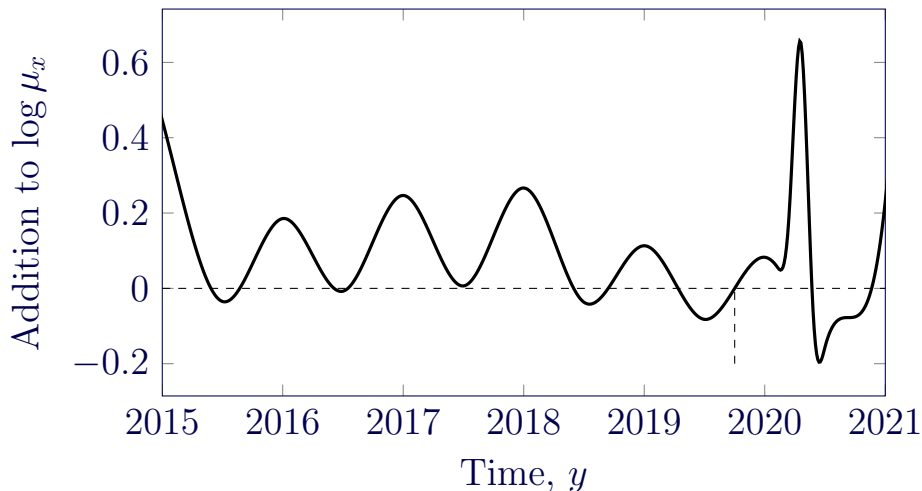


Source: Richards and Macdonald [2024, Figure 3(a)].

- Pension schemes and annuity portfolios are like medical trials:
 - ▶ Continuous recruitment (new retirals, surviving spouses).
 - ▶ Withdrawals/loss to follow-up (transfers out, commutation).
- Binomial models are not well suited to this...
...but survival models are.

Continuous-time modelling gives far greater insight into rapid changes.

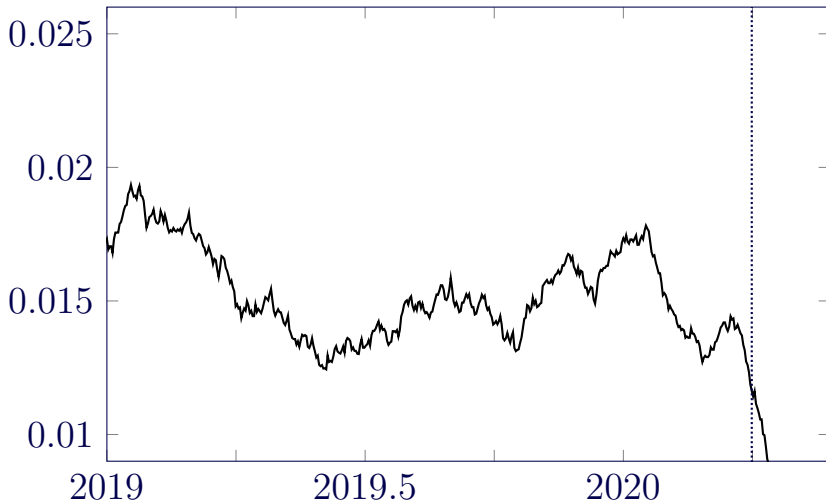
Period effects after allowing for age, gender and pension size:



Source: Richards [2022b, Figure 17(a)].

Benefit 4: Management information

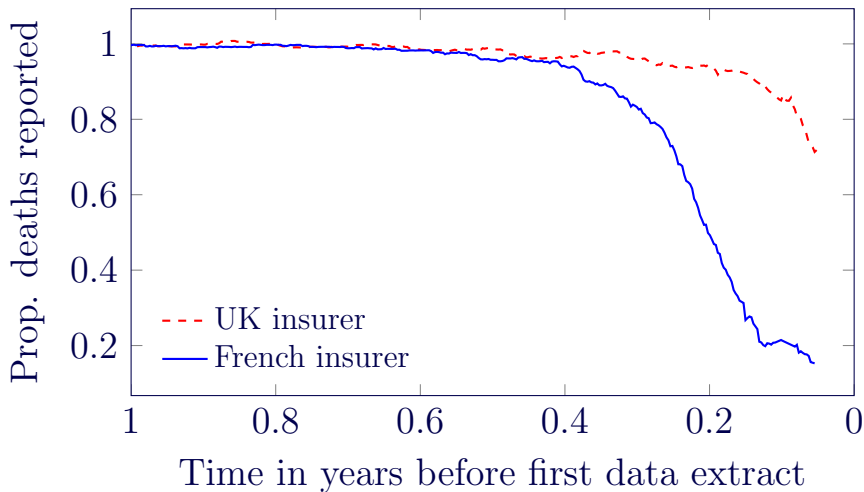
Mortality hazard using June 2020 extract:



Source: Richards and Macdonald [2024, Figure 15(a)].

1. No sign of pandemic mortality by June 2020.
2. Problem of delays in reporting deaths...

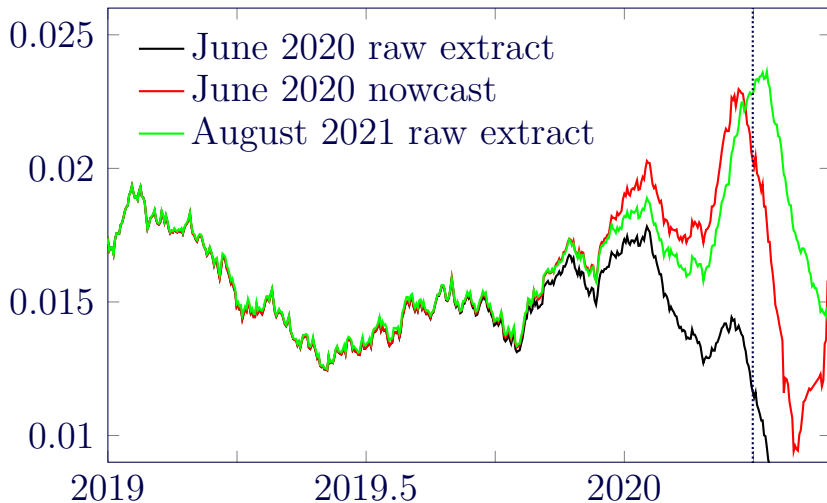
Estimated proportion of deaths reported for two annuity portfolios:



Source: Richards [2022a, Section 4].

1. Estimate the delay function.
2. Use this to “gross up” estimate of current mortality.
3. Bańbura et al. [2013] call this a “nowcast”...

Mortality hazard:



Source: Richards and Macdonald [2024, Figure 15].

3 Conclusions

With continuous-time methods actuaries can:

1. Improve data-quality checking,
2. Match the reality of business processes,
3. Model rapid changes in risk, and
4. Get timelier management information.

- M. Bańbura, D. Giannone, M. Modugno, and L. Reichlin. Chapter 4 — Now-Casting and the Real-Time Data Flow. In Graham Elliott and Allan Timmermann, editors, *Handbook of Economic Forecasting*, volume 2, pages 195–237. Elsevier, 2013. doi: 10.1016/B978-0-444-53683-9.00004-9.
- E. L. Kaplan and P. Meier. Nonparametric estimation from incomplete observations. *Journal of the American Statistical Association*, 53:457–481, 1958.

- S. J. Richards. Real-time measurement of portfolio mortality levels in the presence of shocks and reporting delays. *Annals of Actuarial Science*, 16(3): 430–452, 2022a. doi: 10.1017/S1748499522000021.
- S. J. Richards. Allowing for shocks in portfolio mortality models. *British Actuarial Journal*, 27:1–22 (with discussion), 2022b. doi: 10.1017/S1357321721000180.
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