Longevitas client webinar

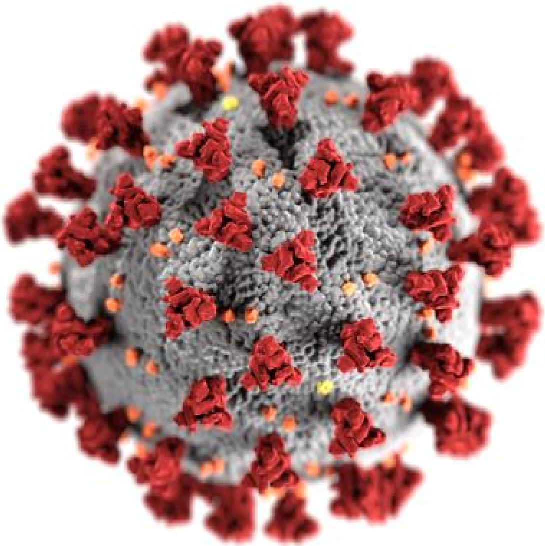
Allowing for shocks in portfolio mortality models

Stephen J. Richards

Wednesday, 1st September 2021, 15:00hrs

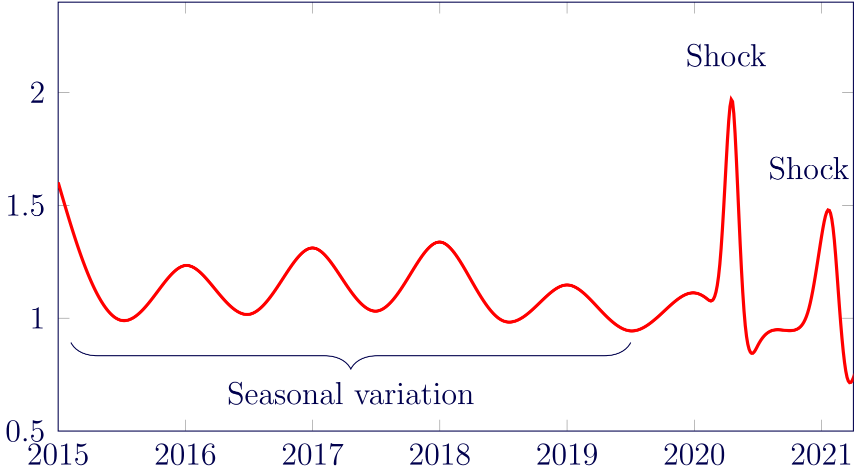
Overview

1 Executive summary



1 Executive summary

Mortality level by time for UK annuity portfolio.



1 Executive summary

- Identify and measure shocks in portfolio data.
- Remove upward bias in mortality analysis for pricing.
- Use all available data, even periods affected by reporting delays.
- BIC is a better measure of fit than the AIC.

1 Executive summary

Presentation based on Richards [2021]:

"Allowing for shocks in portfolio mortality models"

which is freely available at:

www.longevitas.co.uk/site/library/TimeSplines.pdf

2 Motivation

2 Motivation

- Annuities and pensions business.
- Actuaries analyse portfolio experience to set bases.
- Covid-19 mortality spikes in 2020–2021.
- Upward bias in derived mortality levels...

2 Bias is a business problem

Reserving

✗ Imprudent to include recent shock mortality in long-term basis.

Pricing

✗ Under-pricing of bulk annuities and longevity swaps.

2 Some non-solutions

Build a cause-of-death model?

✗ Pension schemes don't record cause of death.

Ignore experience data including shocks?

✗ Often only have data for last 3–5 years.

2 Motivation

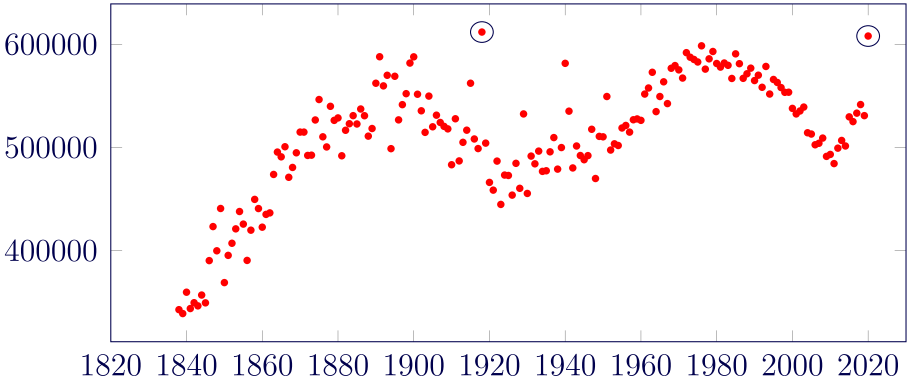
Need a method that:

- Works with available data,
- Works with all data, and
- Handles sharp spikes in mortality.

3 Mortality shocks in UK

LONGEVITAS

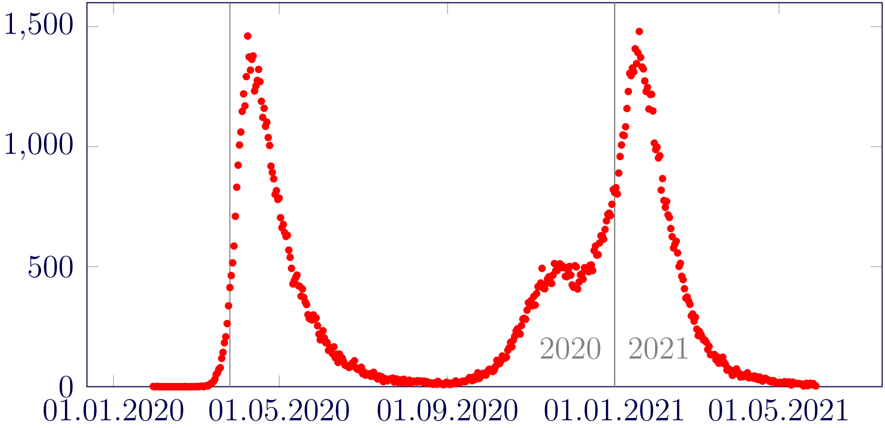
Numbers of deaths in England & Wales (2020 count is provisional).



Source: ONS data.

3 Covid-19, 2020–2021

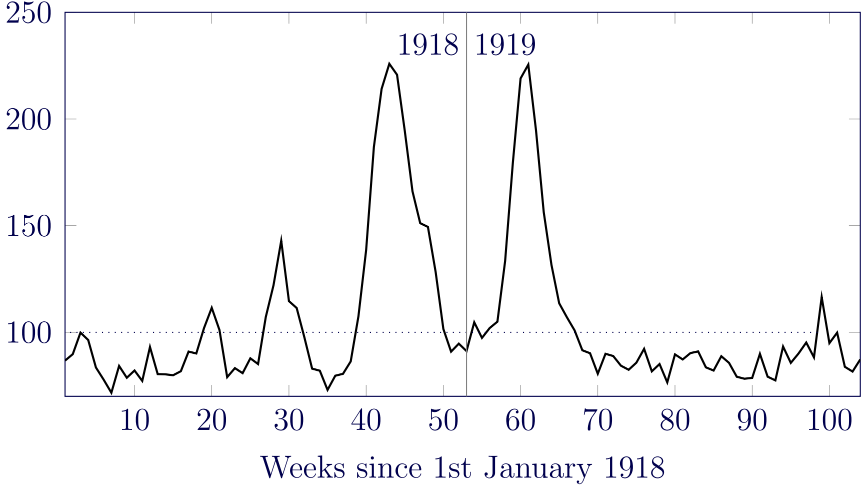
UK deaths where the death certificate mentions covid-19 as one of the causes.



Source: ONS data.

3 Influenza, 1918–1919

Weekly deaths in Scotland as percentage of 1913–1917 average.



Source: Craufurd Dunlop and Watt [1915, 1916a,b, 1918, 1919, 1920a,b].

3 Mortality shocks

- Viral mortality shocks are not new.
- Double spikes in quick succession not new either.
- Need very flexible modelling of mortality in time.

4 Data and features

4 Data extract

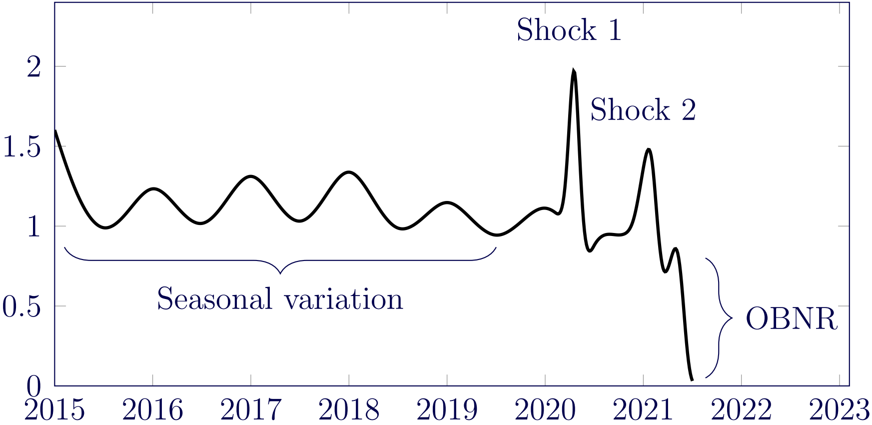
- UK insurer.
- Annuities in payment.
- 351,947 annuities extracted at end-June 2021.
- Policies not independent. . .

4 Deduplication

- Deduplicate to create data set of independent lives.
- 227,527 individuals.
- Average of 1.55 annuities per person.

4 Mortality features

Mortality time index, UK annuity portfolio.



Source: Richards [2021, Figure 18].

4 Mortality features

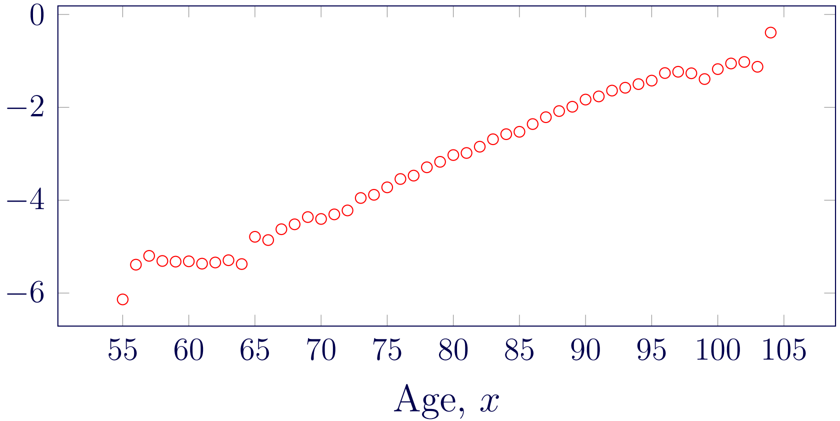
- Strong seasonal variation.
- Pronounced mortality spikes due to covid-19.
- Occurred-but-not-reported (OBNR) deaths†.

† We follow Lawless [1994] in using the term OBNR, as the more familiar term IBNR refers to general insurance claims reserving.

5 Mortality by age and time

5 Mortality by age

log(mortality hazard) for UK3 data set, ages 55–105, 2015–2019.



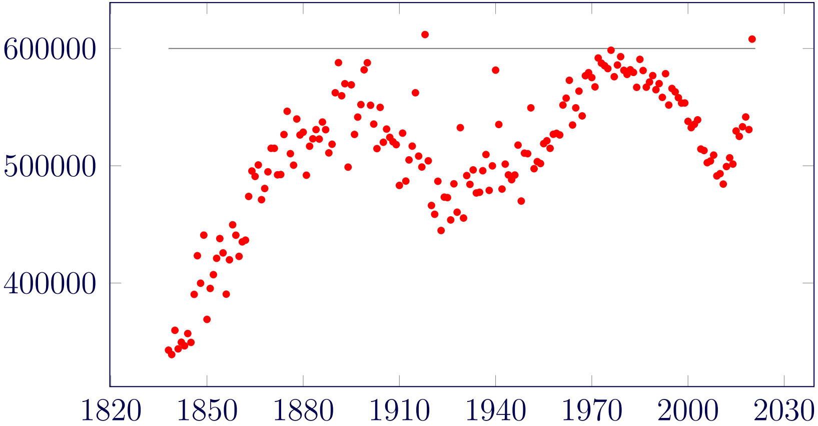
Source: Richards [2021].

5 Mortality by age

- Gradual change over years of age.
- Monotonic increasing.
- Smooth.

LONGEVITAS

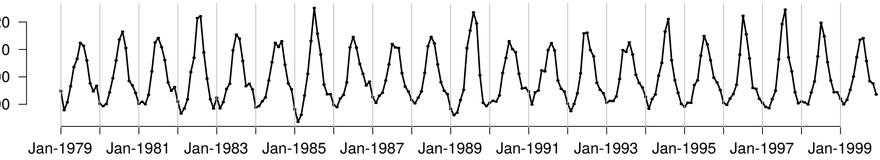
Deaths in England & Wales (2020 count provisional).



Source: ONS.

5 Intra-year mortality: seasons

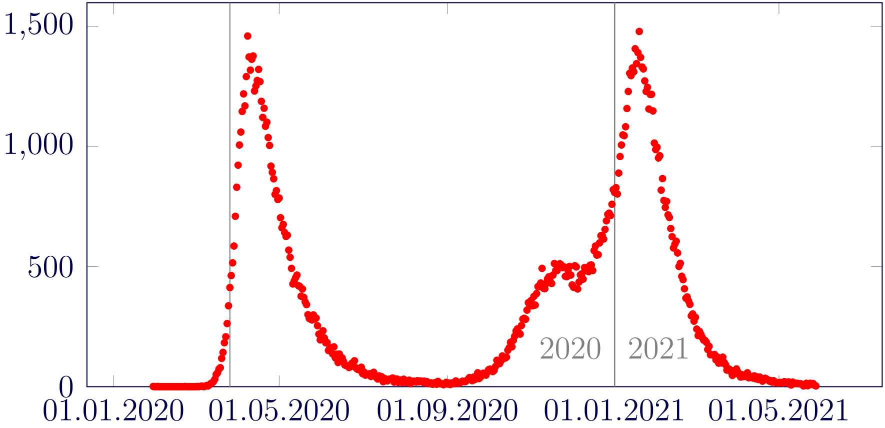
Percentage of average daily number of deaths in Australia, all causes, 1979–1999.



Source: de Looper [2002].

5 Intra-year mortality: shocks

UK deaths where the death certificate mentions COVID-19 as one of the causes.



Source: ONS data.

5 Mortality in time

- Not monotonic (ever).
- Not smooth on a year-to-year basis…

  …but smooth on a day-to-day basis.

  (even during a pandemic)

5 Modelling requirements

Mortality by age

Slow, monotonic changes need little flexibility.

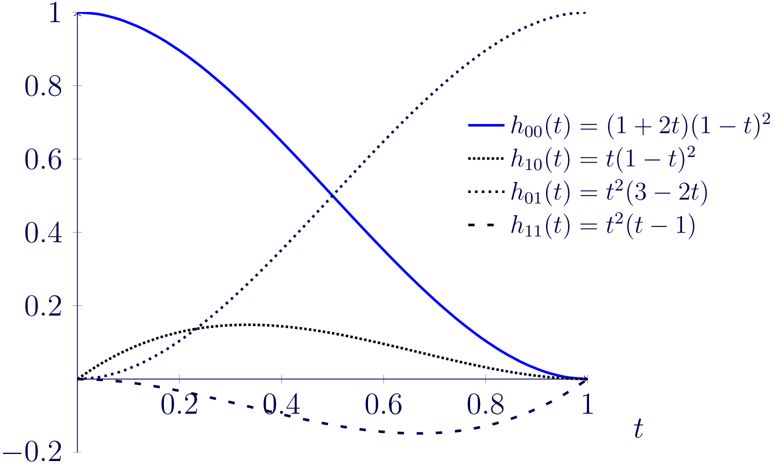
Mortality by time

Fast, non-monotonic changes need greater flexibility

$\rightarrow$ Split model into separate age and time components.

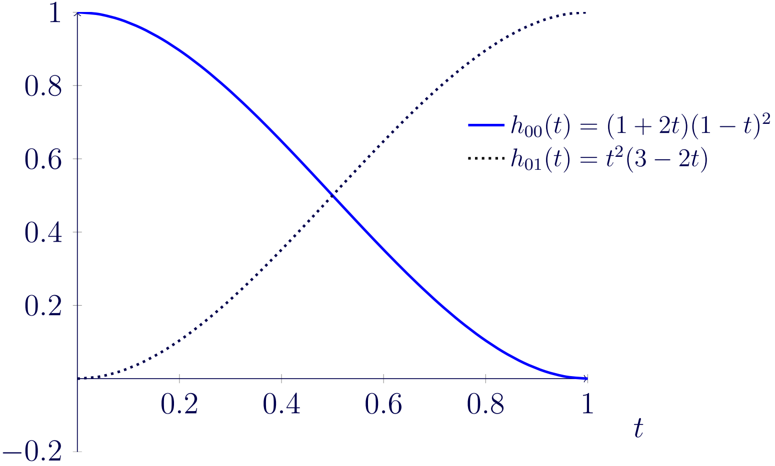
6 Age component

6 A basis of Hermite splines



Source: Richards [2020].

6 Sub-basis of Hermite splines



Source: Richards [2020].

6 Hermite-spline model

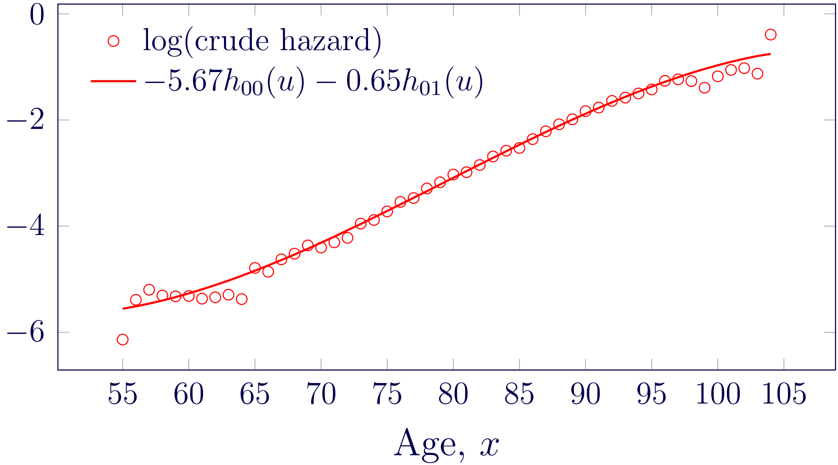
- x_0 is minimum age.
- x_1 is maximum age.
- Define $u = \frac{(x - x_0)}{(x_1 - x_0)}$, so $u \in [0, 1]$.
- $\log \mu_x = \alpha h_{00}(u) + \omega h_{01}(u)$

for parameters α and ω estimated from data.

Source: Richards [2020].

6 Hermite-spline model

log(mortality hazard) (∘) for UK3 data set with fitted curve (—) comprising two of the Hermite basis splines.



6 Age component

A two-parameter Hermite-spline model is often enough for mortality by age.

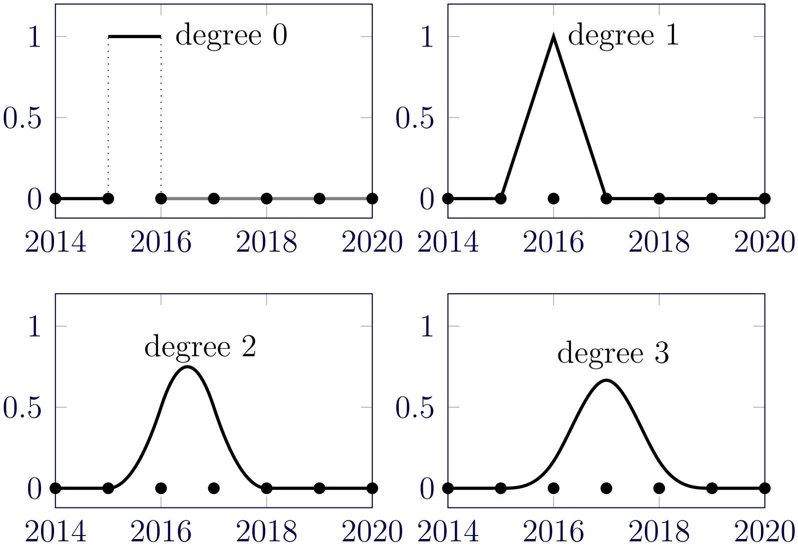
6 Hermite-spline model

Note that $h_{00}(u) + h_{01}(u) = 1$, so...

$$\begin{aligned}\log \mu_x &= \alpha h_{00}(u) + \omega h_{01}(u) \\ &= \alpha h_{00}(u) + \omega h_{01}(u) + c - c \\ &= (\alpha + c)h_{00}(u) + (\omega + c)h_{01}(u) - c\end{aligned}$$

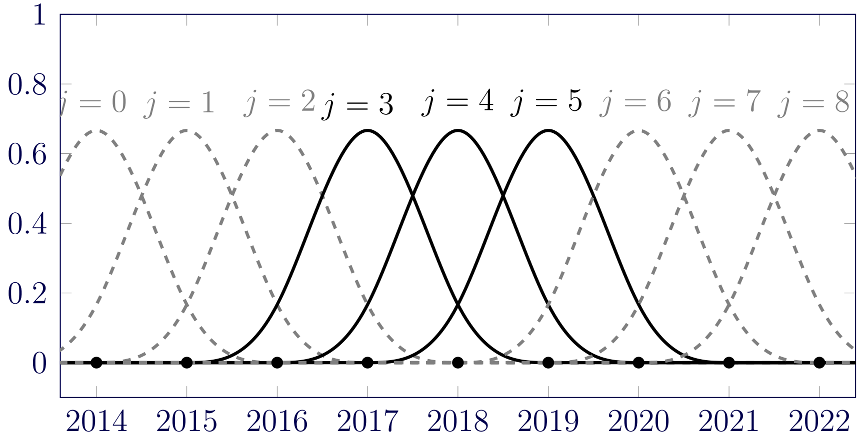
7 Time component

7 Schoenberg [1964] splines



7 A basis of cubic B-splines

A basis of nine equally-spaced cubic B-splines spanning 1st January 2015 to end-2020, indexed $j = 0, 1, \ldots, 8$.



7 B-splines

- Define $B_j(y)$ as the j^{th} basis spline at time y.
- Then $\sum_{j\geq 0} B_j(y) = 1, \forall y \in [2015, 2021]$.
- And $\sum_{j\geq 0} cB_j(y) = c, \forall y \in [2015, 2021]$ and $c \in \mathbb{R}$.

Define:

- μ_x, the Hermite-spline model for mortality by age.
- $\mu_{x,y}$, the mortality hazard at age x and time y.
- $\kappa_{0,j}$, the coefficient of spline B_j.

7 Continuous age-period model

$$\log \mu_{x,y} = \underbrace{\log \mu_x}_{\text{Hermite age component}} + \underbrace{\sum_{j \geq 1} \kappa_{0,j} B_j(y)}_{\text{Schoenberg time component}}$$

- Why summation from $j = 1$ and not $j = 0$?
- Need identifiability constraint.
- Use $\kappa_{0,0} = 0$ for simplicity.

7 Estimates of $\kappa_{0,j}$

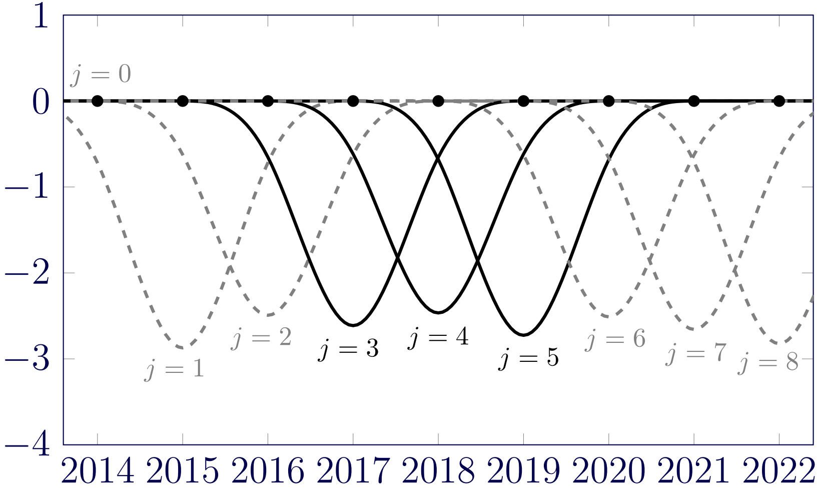
$\hat{\kappa}_{0,j}$ for $j = 1, 2, \ldots, 8$ for UK3 portfolio, 2015 to end-2020.

j	1	2	3	4	5	6	7	8
$\hat{\kappa}_{0,j}$	-4.30805	–3.73912	-3.91987	-3.69781	-4.0887	-3.76673	-3.98538	-4.23435

$\kappa_{0,0} = 0$ by construction because it is absorbed into the baseline hazard.

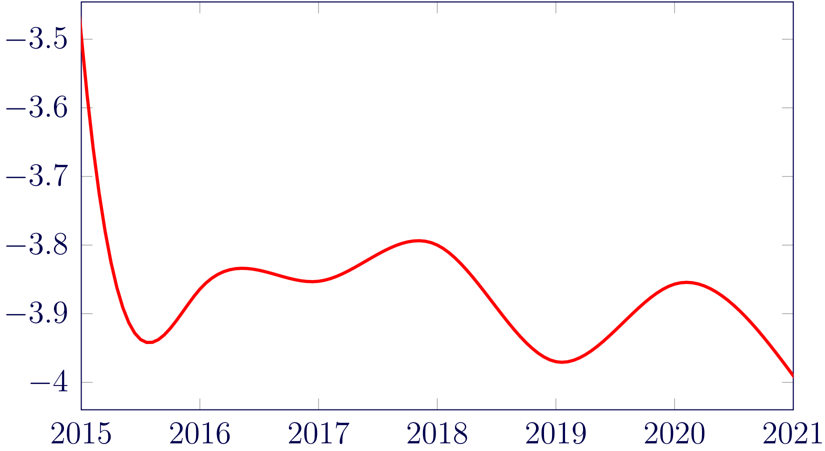
7 Effect of $\hat{\kappa}_{0,j}$

$$\hat{\kappa}_{0,j} B_j(y).$$



7 Combining $\hat{\kappa}_{0,j}$

$\sum_{j\geq 1} \hat{\kappa}_{0,j} B_j(y)$ for y spanning 1st January 2015 to end-2020.



7 Normalising

- Vertical scale with $\kappa_{0,0} = 0$ is somewhat arbitrary.
- Can use other identifiability constraints.
- Can deduct $c \in \mathbb{R}$ from every $\kappa_{0,j}$ as long as c is added to $\log \mu_x$.

7 Normalising

$$
\begin{aligned}
&\alpha h_{00}(u) + \omega h_{01}(u) + \sum_{j\geq 0} \kappa_{0,j} B_j(y) \\
&= \alpha h_{00}(u) + \omega h_{01}(u) + c - c + \sum_{j\geq 0} \kappa_{0,j} B_j(y) \\
&= \alpha h_{00}(u) + \omega h_{01}(u) + c - \sum_{j\geq 0} c B_j(y) + \sum_{j\geq 0} \kappa_{0,j} B_j(y) \\
&= (\alpha + c) h_{00}(u) + (\omega + c) h_{01}(u) + \sum_{j\geq 0} (\kappa_{0,j} - c) B_j(y)
\end{aligned}
$$

7 Normalising

What if we normalise at zero on 1st October 2019, i.e. mid-way between last summer trough and winter peak before covid-19?

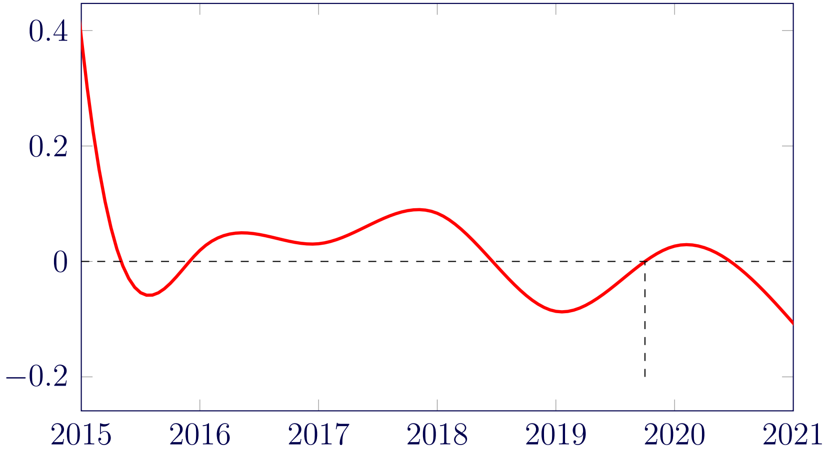
7 Normalising

- Calculate $c_{2019.75} = \sum_{j \geq 1} \hat{\kappa}_{0,j} B_j(2019.75)$.
- Re-balance with:
  - $\sum_{j \geq 0} (\hat{\kappa}_{0,j} - c_{2019.75}) B_j(y)$,
  - $\alpha' = \alpha + c_{2019.75}$, and
  - $\omega' = \omega + c_{2019.75}$.

…and the model fit is unchanged.

7 Combining $\hat{\kappa}_{0,j}$

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y) \text{ for } y \in [2015, 2021]$$



7 Adding TimeSpline term

TimeSpline option available for all Hermite models:

Term Groups To Include

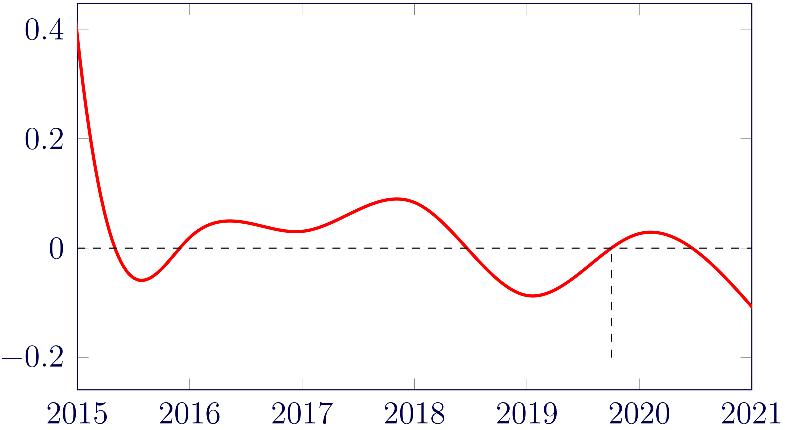
	Term Group	Fixed Terms	Optional Terms
Include	☐ AgeTimeTrend	TrendPeak TrendPeakAge	☐ TrendYoungest
	☐ Selection	SelectionInitial SelectionTerm	☐ SelectionGradient
	☐ Season	SeasonalExcess SeasonalPeak	☐ SeasonalAge
	☐ Amount	AmountTransformParameter AmountUltimate	☐ AmountGradientInitial ☐ AmountGradientUltimate
	☐ OBNR	OBNRdecay	
	☑ TimeSpline	TimeSpline	

8 Seasons and shocks

- Previous slides used one-year knot spacing.
- What if we use half-year knot spacing?
- Or quarter-year knot spacing?

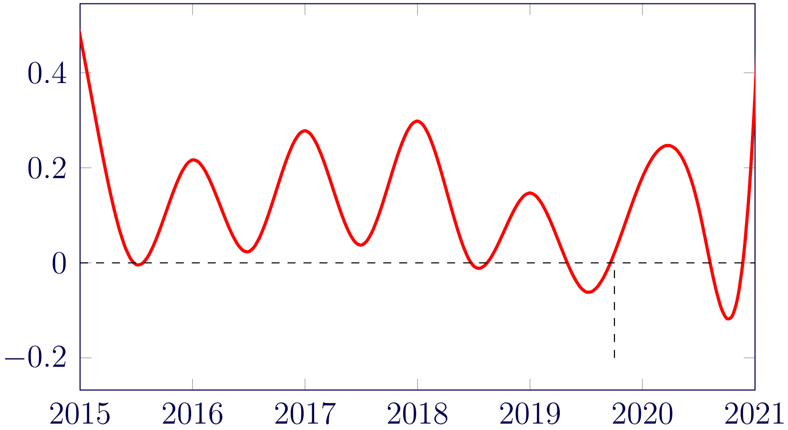
8 UK3, one knot per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



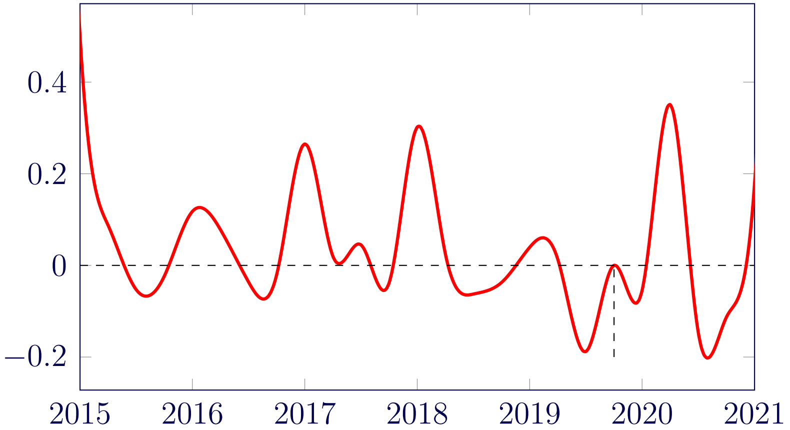
8 UK3, two knots per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



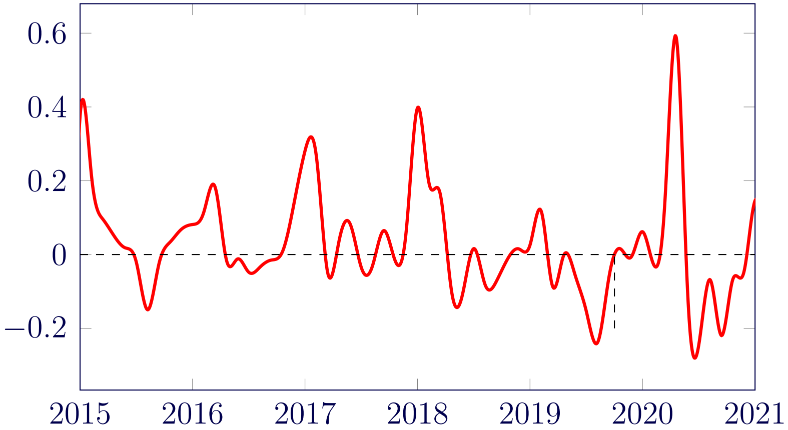
8 UK3, four knots per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



8 UK3, ten knots per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



8 Seasons and shocks

- Half-year knot spacing reveals seasonal variation.
- 4 and 10 knots per year reveal covid-19 shock...

  ...but also introduce random variation pre-shock.

8 Measuring fit

Knots per year	Parameter count	AIC	BIC
1	14	187,594	187,729
2	20	187,412	187,605
4	32	187,324	187,634
10	68	187,244	187,901

Source: Richards [2021, Table 4].

8 Contradiction!

- AIC lowest with 10 knots per year.
- BIC lowest with 2 knots per year.
- AIC under-penalises parameters…

  ….and leads to over-parameterisation.

8 Contradiction!

- This is not about the small-sample correction to the AIC [Hurvich and Tsai, 1989]
  ($n = 116,056$, so sample is not small!)
- Nor is this about a large parameter-to-observation ratio.
- Issue appears to be about number of degrees of freedom used when many parameters are insignificant; see discussion in Richards [2021, Section 12].

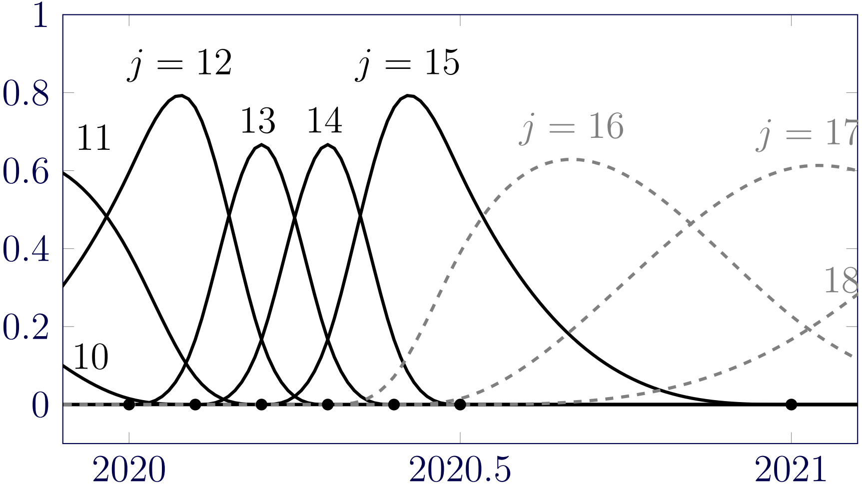
8 Variable knot spacing

- Knots don't have to be equally spaced [Kaishev et al., 2016].
- Use two knots per year for seasonal variation...

  ...and add knots where we know the shocks are.

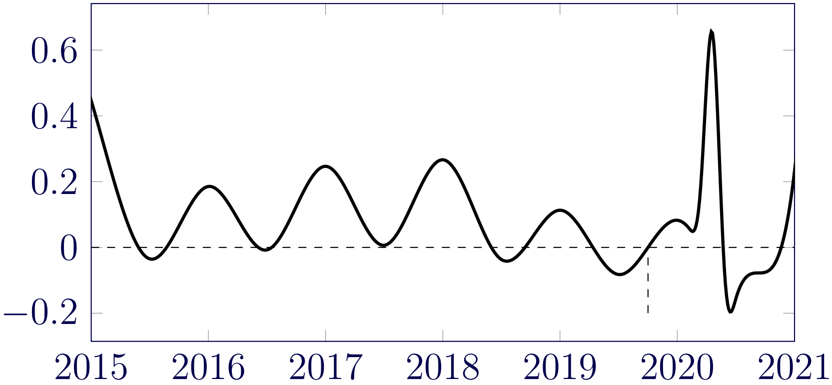
8 Variable knot spacing

Part of a basis of nineteen variably-spaced cubic B-splines.



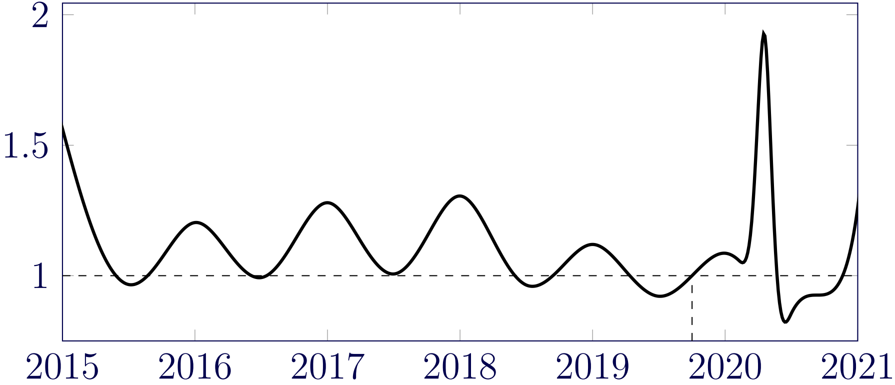
8 Variable knot spacing

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



8 Variable knot spacing

$$\exp\left(\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)\right)$$

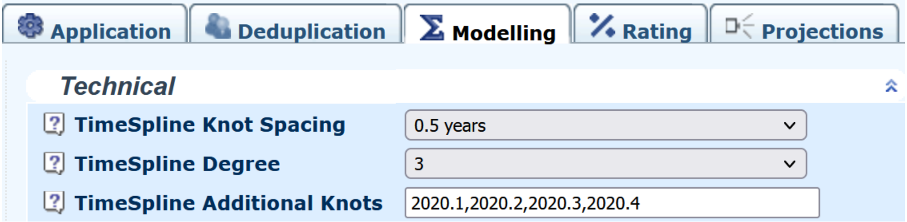


8 Variable knot spacing

- Seasonal variation means peak winter mortality is 15–30% higher than summer mortality.
- Mortality hazard doubled in April-May 2020 relative to baseline of October 2019.

8 Knot control

Configuration for basic knot spacing, spline degree and hand-placed knots:



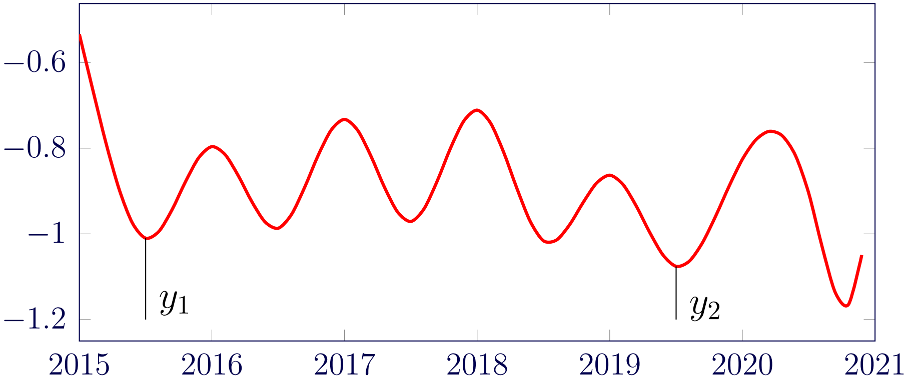
9 Mortality improvements

9 Mortality improvements

- We can also estimate portfolio-specific mortality improvements.
- Consider time component at y_1 v. y_2.
- Use midsummer points for stability.

9 UK3, two knots per year

$$\sum_{j\geq 1} \hat{\kappa}_{0,j} B_j(y) \text{ for } y \in [2015, 2021].$$



Annual improvement rate, i, between y_1 and y_2:

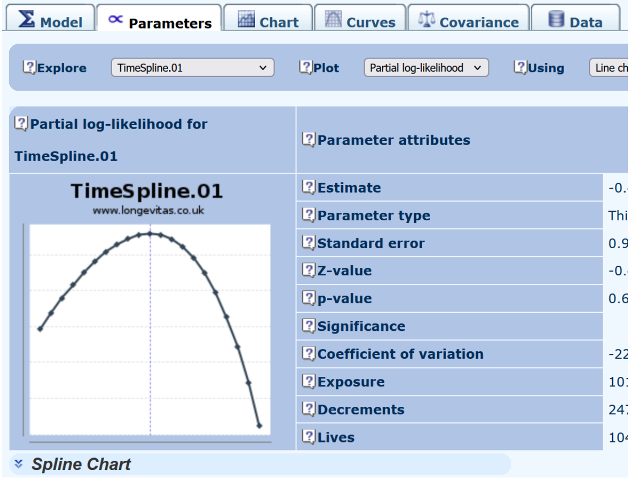
$$i_{y_1,y_2} = \left[1 - \exp\left(\frac{\sum_{j\geq 1} \hat{\kappa}_{0,j}\left[B_j(y_2) - B_j(y_1)\right]}{y_2 - y_1}\right)\right] \times 100\%$$

Source: Richards [2021].

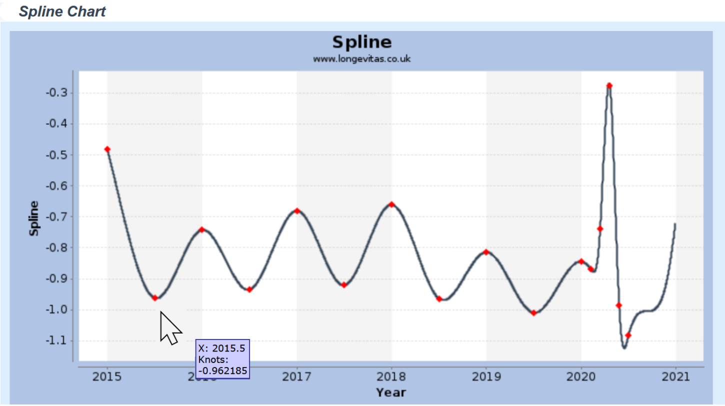
- For UK3 aggregate annual improvement rate between mid-2015 and mid-2019 was 1.2% p.a.
- Can compare with CMI model used for reserving.

9 Sourcing $\sum_{j\geq 1} \hat{\kappa}_{0,j} B_j(y)$

i) Select any TimeSpline parameter in **Parameter** tab:



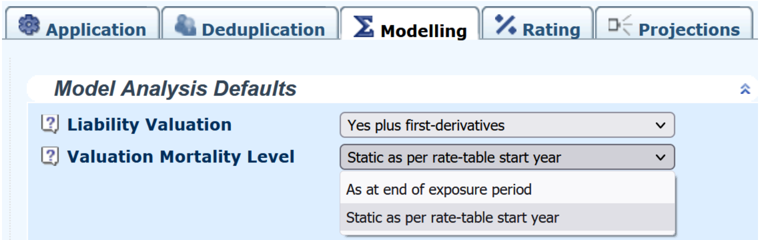
ii) Open **Spline Chart** and use mouse to read off value:



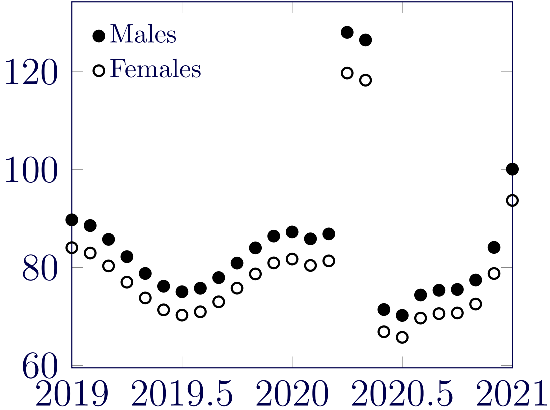
10 Valuation

10 Valuation control

- Valuation of benefits paid to survivors at end of the data period.
- However, valuation mortality levels can be:
  1. those at the end of the data period, or
  2. those at an earlier point in the data period.



Percentages of S2PA implied by mortality levels over 2019–2020:



Source: Richards [2021, Figure 22].

11 Conclusions

11 Conclusions — I

Modelling by age needs little flexibility

Use Hermite splines.

Modelling in time needs lots of flexibility

Use Schoenberg [1964] splines.

11 Conclusions — II

- Add knots around pandemic shocks.
- BIC better than AIC for model selection.
- Exercise judgement as to normal mortality level.
- Can estimate portfolio-specific improvement rate.

References I

J. C. Craufurd Dunlop and A. Watt. *Fifty-ninth annual report of the Registrar General for Scotland*, volume 59. H.M. Stationery Office, Glasgow, 1915.

J. C. Craufurd Dunlop and A. Watt. *Sixtieth annual report of the Registrar General for Scotland*, volume 60. H.M. Stationery Office, Glasgow, 1916a.

J. C. Craufurd Dunlop and A. Watt. *Sixty-first annual report of the Registrar General for Scotland*, volume 61. H.M. Stationery Office, Glasgow, 1916b.

J. C. Craufurd Dunlop and A. Watt. *Sixty-second annual report of the Registrar General for Scotland*, volume 62. H.M. Stationery Office, Edinburgh, 1918.

References II

J. C. Craufurd Dunlop and A. Watt. *Sixty-third annual report of the Registrar General for Scotland*, volume 63. H.M. Stationery Office, Edinburgh, 1919.

J. C. Craufurd Dunlop and A. Watt. *Sixty-fourth annual report of the Registrar General for Scotland*, volume 64. H.M. Stationery Office, Edinburgh, 1920a.

J. C. Craufurd Dunlop and A. Watt. *Sixty-fifth annual report of the Registrar General for Scotland*, volume 65. H.M. Stationery Office, Edinburgh, 1920b.

References III

M. de Looper. *Seasonality of death*, volume Bulletin No. 3. Australian Institute of Health and Welfare, 2002. ISBN 978–1–74024–209–7.

C. M. Hurvich and C. L. Tsai. Regression and time-series model selection in small samples. *Biometrika*, 76(2):297–307, 1989.

V. K. Kaishev, D. S. Dimitrova, S. Haberman, and R. J. Verrall. Geometrically designed, variable knot regression splines. *Computational Statistics*, 31(3): 1079–1105, 2016. doi: 10.1007/s00180-015-0621-7.

References IV

J. F. Lawless. Adjustments for reporting delays and the prediction of occurred but not reported events. *Canadian Journal of Statistics*, 22(1):15–31, 1994. doi: https://doi.org/10.2307/3315826.n1.

S. J. Richards. A Hermite-spline model of post-retirement mortality. *Scandinavian Actuarial Journal*, 2020:2:110–127, 2020. doi: 10.1080/03461238.2019.1642239.

S. J. Richards. Allowing for shocks in portfolio mortality models. *Longevitas Ltd*, 2021.

References V

I. J. Schoenberg. Spline functions and the problem of graduation. *Proceedings of the American Mathematical Society*, 52:947–950, 1964.

Coronavirus graphic  from CDC

More on longevity risk at www.longevitas.co.uk

12 Legal matters

Longevitas is a registered trademark of Longevitas Ltd:

- in the UK (No. 2434941),
- in the USA (No. 3707314), and
- in the European Union (No. 5854518).