Longevity 16, Copenhagen, Denmark

Allowing for shocks in portfolio mortality models

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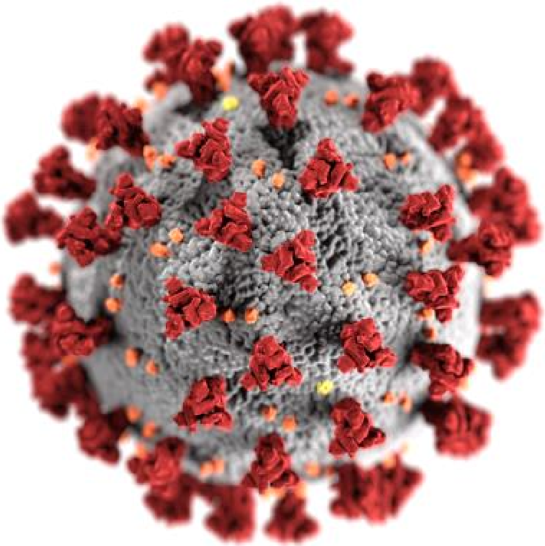
Overview

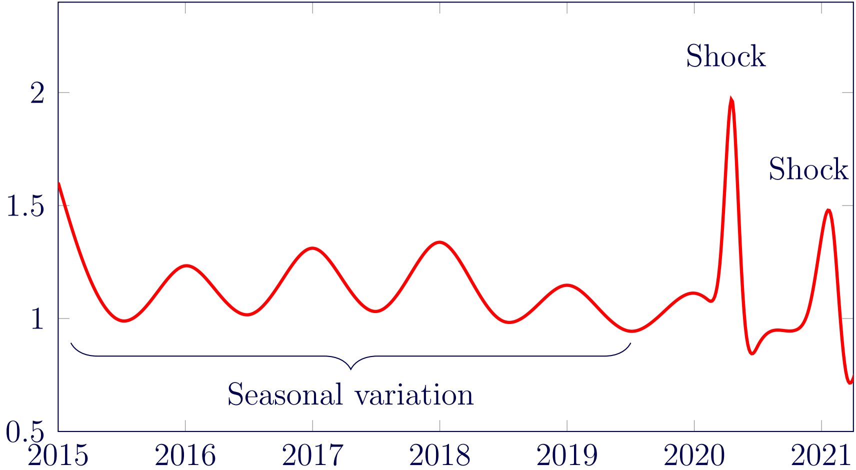
1. Executive summary
2. Motivation
3. Mortality shocks in UK
4. Data and features
5. Mortality by age and time
6. Age component
7. Time component
8. Seasons and shocks
9. Mortality improvements
10. Conclusions

1 Executive summary



1 Executive summary

Mortality level by time for UK annuity portfolio.



1 Executive summary

- Identify and measure shocks in portfolio data.
- Remove upward bias in mortality analysis for pricing.
- Use all available data, even periods affected by reporting delays.
- BIC is a better measure of fit than the AIC.

1 Executive summary

Presentation based on Richards [2021]:

"Allowing for shocks in portfolio mortality models"

which is freely available at:

www.longevitas.co.uk/site/library/TimeSplines.pdf

2 Motivation

2 Motivation

- Annuities and pensions business.
- Actuaries analyse portfolio experience to set bases.
- Covid-19 mortality spikes in 2020–2021.
- Upward bias in derived mortality levels…

2 Bias is a business problem

Reserving

✗ Imprudent to include recent shock mortality in long-term basis.

Pricing

✗ Under-pricing of bulk annuities and longevity swaps.

2 Some non-solutions

Build a cause-of-death model?

✗ Pension schemes don’t record cause of death.

Ignore experience data including shocks?

✗ Often only have data for last 3–5 years.

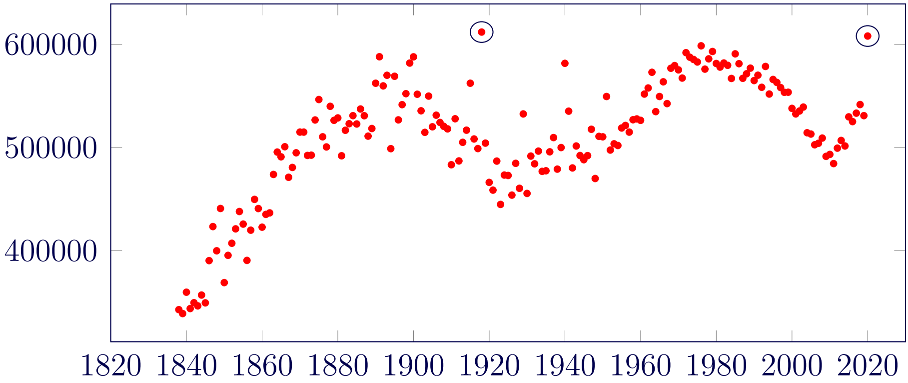
2 Motivation

Need a method that:

- Works with available data,
- Works with all data, and
- Handles sharp spikes in mortality.

3 Mortality shocks in UK

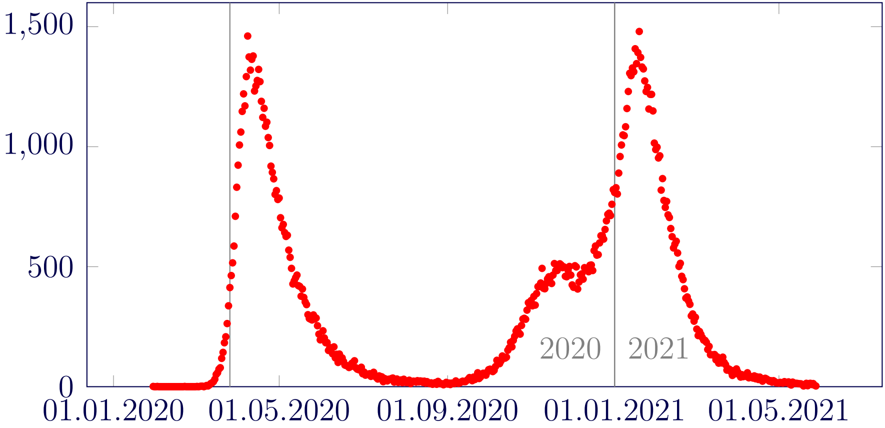
Numbers of deaths in England & Wales (2020 count is provisional).



Source: ONS data.

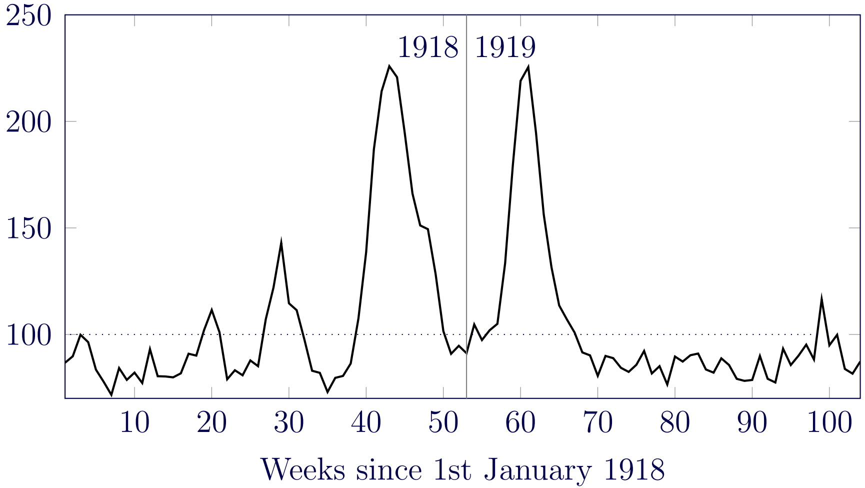
3 Covid-19, 2020–2021

UK deaths where the death certificate mentions covid-19 as one of the causes.



Source: ONS data.

Weekly deaths in Scotland as percentage of 1913–1917 average.



Source: Craufurd Dunlop and Watt [1915, 1916a,b, 1918, 1919, 1920a,b].

3 Mortality shocks

- Viral mortality shocks are not new.
- Double spikes in quick succession not new either.
- Need very flexible modelling of mortality in time.

4 Data and features

4 Data extract

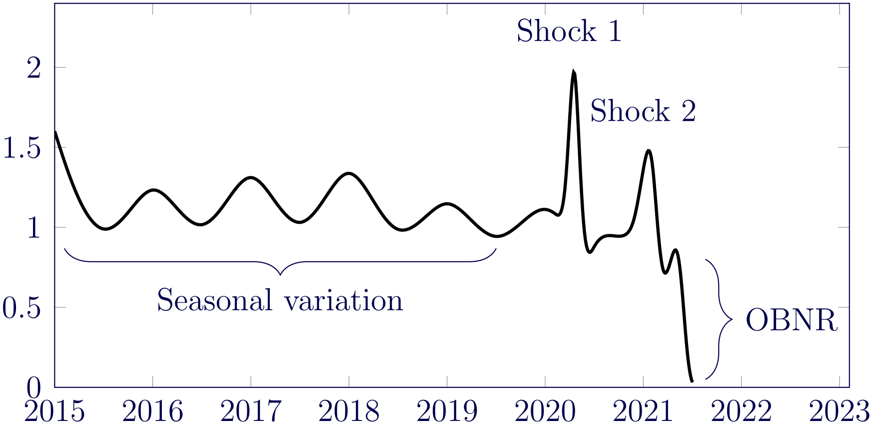
- UK insurer.
- Annuities in payment.
- 351,947 annuities extracted at end-June 2021.
- Policies not independent. . .

4 Deduplication

- Deduplicate to create data set of independent lives.
- 227,527 individuals.
- Average of 1.55 annuities per person.

4 Mortality features

Mortality time index, UK annuity portfolio.



Source: Richards [2021, Figure 18].

4 Mortality features

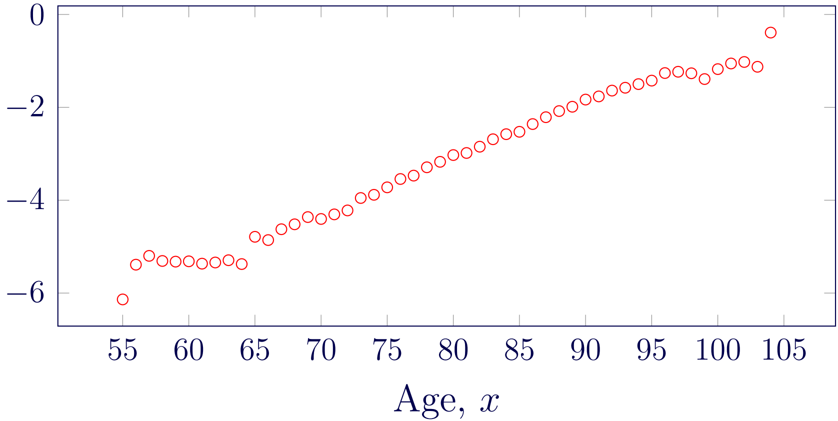
- Strong seasonal variation.
- Pronounced mortality spikes due to covid-19.
- Occurred-but-not-reported (OBNR) deaths†.

† We follow Lawless [1994] in using the term OBNR, as the more familiar term IBNR refers to general insurance claims reserving.

5 Mortality by age and time

5 Mortality by age

log(mortality hazard) for UK3 data set, ages 55–105, 2015–2019.

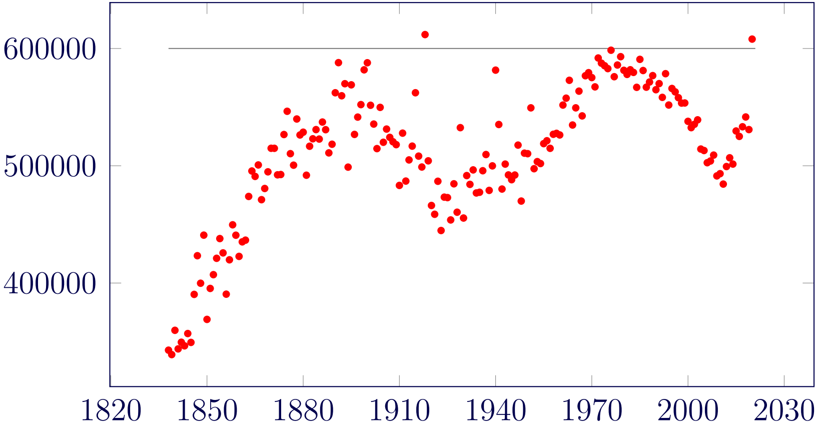


Source: Richards [2021].

5 Mortality by age

- Gradual change over years of age.
- Monotonic increasing.
- Smooth.

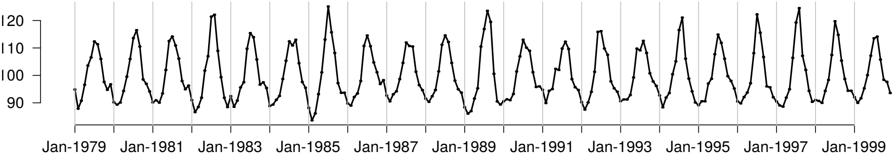
Deaths in England & Wales (2020 count provisional).



Source: ONS.

5 Intra-year mortality: seasons

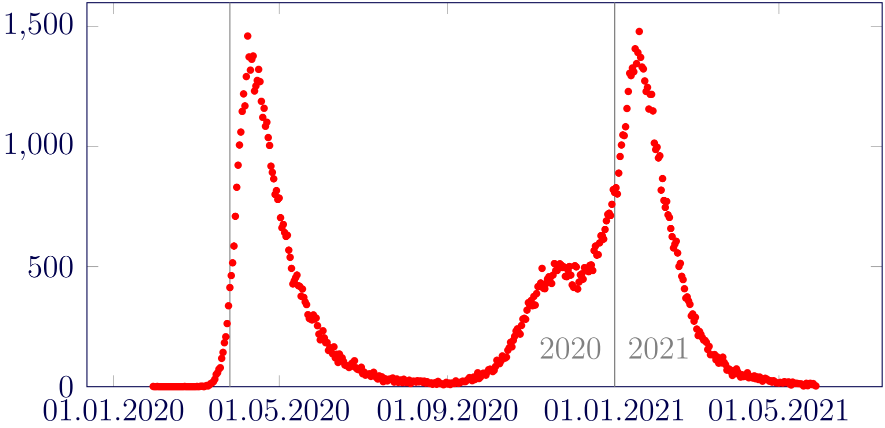
Percentage of average daily number of deaths in Australia, all causes, 1979–1999.



Source: de Looper [2002].

5 Intra-year mortality: shocks

UK deaths where the death certificate mentions COVID-19 as one of the causes.



Source: ONS data.

5 Mortality in time

- Not monotonic (ever).
- Not smooth on a year-to-year basis...

  ...but smooth on a day-to-day basis.

  (even during a pandemic)

5 Modelling requirements

Mortality by age

Slow, monotonic changes need little flexibility.

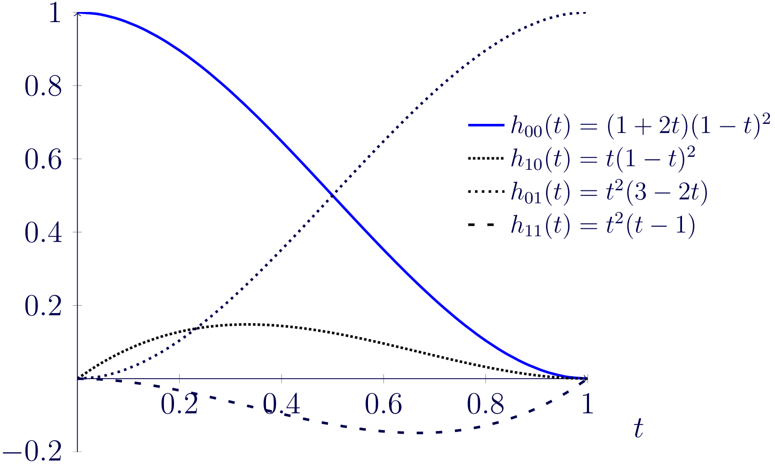
Mortality by time

Fast, non-monotonic changes need greater flexibility

$\rightarrow$ Split model into separate age and time components.

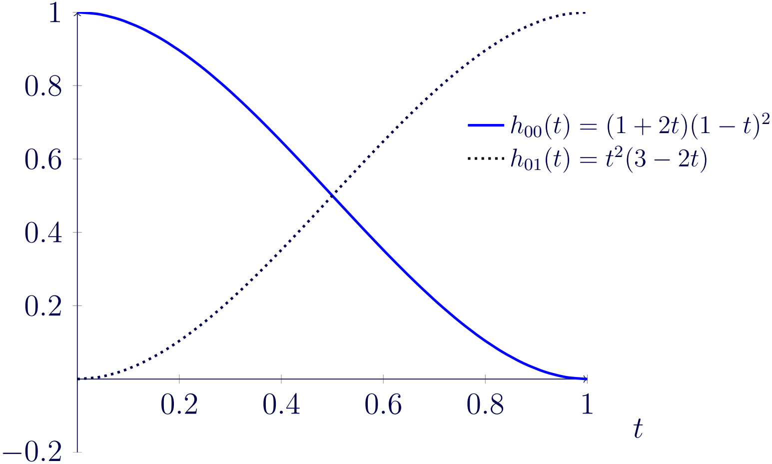
6 Age component

6 A basis of Hermite splines



Source: Richards [2020].

6 Sub-basis of Hermite splines



Source: Richards [2020].

6 Hermite-spline model

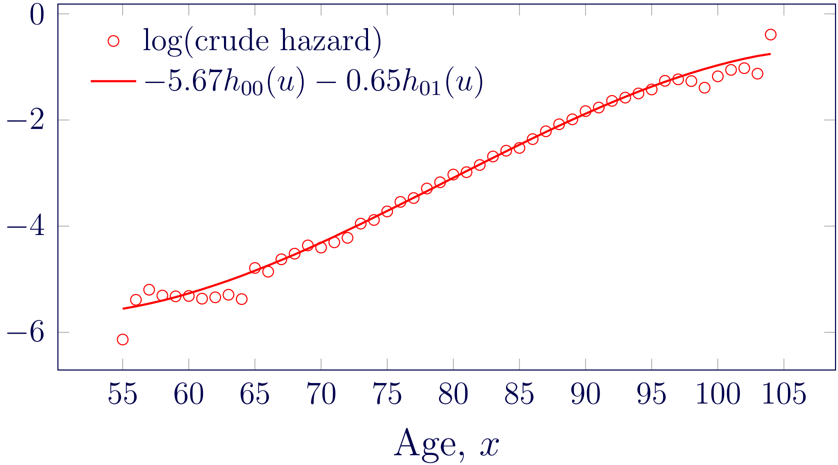
- x_0 is minimum age.
- x_1 is maximum age.
- Define $u = \dfrac{(x - x_0)}{(x_1 - x_0)}$, so $u \in [0, 1]$.
- $\log \mu_x = \alpha h_{00}(u) + \omega h_{01}(u)$

for parameters α and ω estimated from data.

Source: Richards [2020].

6 Hermite-spline model

log(mortality hazard) (∘) for UK3 data set with fitted curve (—) comprising two of the Hermite basis splines.



6 Age component

A two-parameter Hermite-spline model is often enough for mortality by age.

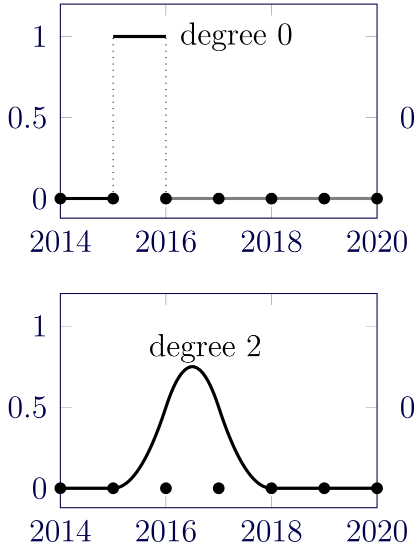
6 Hermite-spline model

Note that $h_{00}(u) + h_{01}(u) = 1$, so...

$$\begin{aligned} \log \mu_x &= \alpha h_{00}(u) + \omega h_{01}(u) \\ &= \alpha h_{00}(u) + \omega h_{01}(u) + c - c \\ &= (\alpha + c) h_{00}(u) + (\omega + c) h_{01}(u) - c \end{aligned}$$

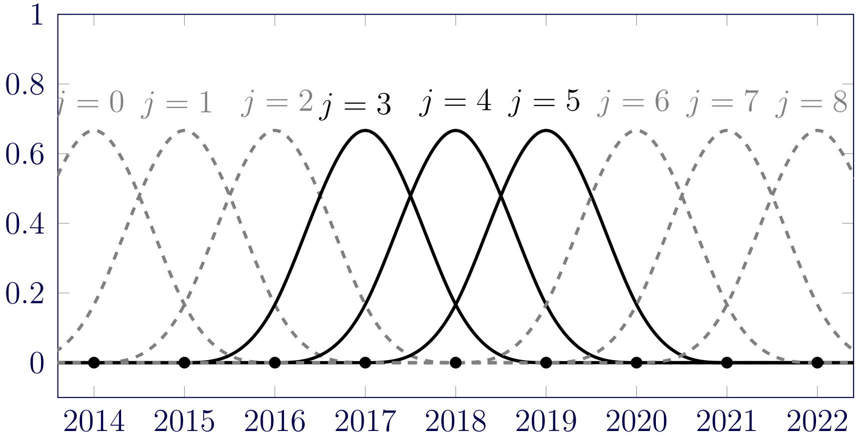
7 Time component

7 Schoenberg [1964] splines



7 A basis of cubic B-splines

A basis of nine equally-spaced cubic B-splines spanning 1st January 2015 to end-2020, indexed $j = 0, 1, \ldots, 8$.



- Define $B_j(y)$ as the j^{th} basis spline at time y.
- Then $\sum_{j \geq 0} B_j(y) = 1, \forall y \in [2015, 2021]$.
- And $\sum_{j \geq 0} cB_j(y) = c, \forall y \in [2015, 2021]$ and $c \in \mathbb{R}$.

7 Model by age and time

Define:

- μ_x, the Hermite-spline model for mortality by age.
- $\mu_{x,y}$, the mortality hazard at age x and time y.
- $\kappa_{0,j}$, the coefficient of spline B_j.

7 Continuous age-period model

$$\underbrace{\log \mu_{x,y} = \log \mu_x}_{\text{Hermite age component}} + \underbrace{\sum_{j\geq 1} \kappa_{0,j} B_j(y)}_{\text{Schoenberg time component}}$$

7 Continuous age-period model

- Why summation from $j = 1$ and not $j = 0$?
- Need identifiability constraint.
- Use $\kappa_{0,0} = 0$ for simplicity.

7 Estimates of $\kappa_{0,j}$

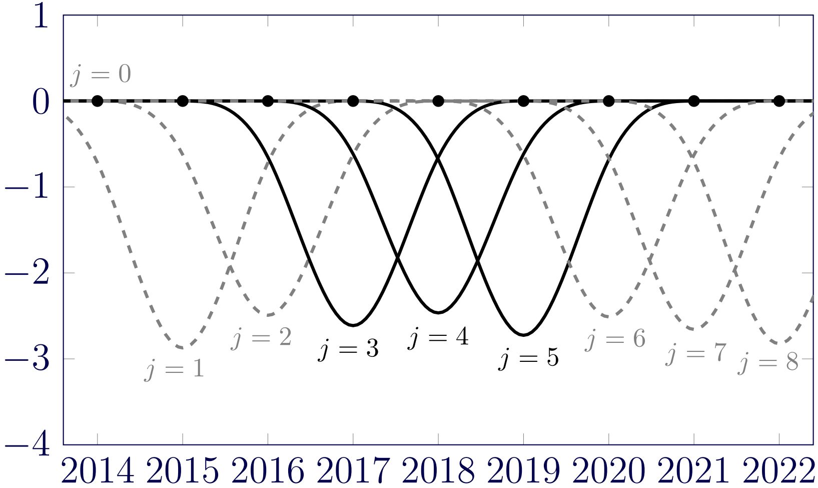
$\hat{\kappa}_{0,j}$ for $j = 1, 2, \ldots, 8$ for UK3 portfolio, 2015 to end-2020.

j	1	2	3	4	5	6	7	8
$\hat{\kappa}_{0,j}$	-4.30805	–3.73912	-3.91987	-3.69781	-4.0887	-3.76673	-3.98538	-4.23435

$\kappa_{0,0} = 0$ by construction because it is absorbed into the baseline hazard.

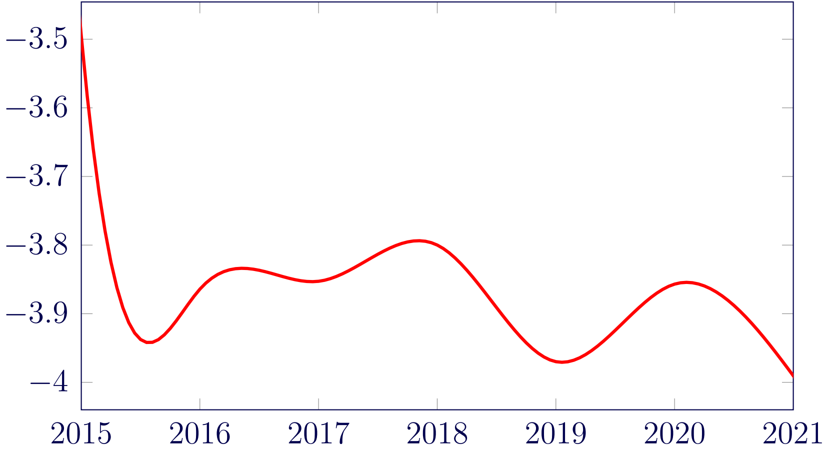
7 Effect of $\hat{\kappa}_{0,j}$

$$\hat{\kappa}_{0,j} B_j(y).$$



7 Combining $\hat{\kappa}_{0,j}$

$\sum_{j \geq 1} \hat{\kappa}_{0,j} B_j(y)$ for y spanning 1st January 2015 to end-2020.



7 Normalising

- Vertical scale with $\kappa_{0,0} = 0$ is somewhat arbitrary.
- Can use other identifiability constraints.
- Can deduct $c \in \mathbb{R}$ from every $\kappa_{0,j}$ as long as c is added to $\log \mu_x$.

7 Normalising

$$
\begin{aligned}
&\alpha h_{00}(u) + \omega h_{01}(u) + \sum_{j\geq 0} \kappa_{0,j} B_j(y) \\
&= \alpha h_{00}(u) + \omega h_{01}(u) + c - c + \sum_{j\geq 0} \kappa_{0,j} B_j(y) \\
&= \alpha h_{00}(u) + \omega h_{01}(u) + c - \sum_{j\geq 0} c B_j(y) + \sum_{j\geq 0} \kappa_{0,j} B_j(y) \\
&= (\alpha + c) h_{00}(u) + (\omega + c) h_{01}(u) + \sum_{j\geq 0} (\kappa_{0,j} - c) B_j(y)
\end{aligned}
$$

What if we normalise at zero on 1st October 2019, i.e. mid-way between last summer trough and winter peak before covid-19?

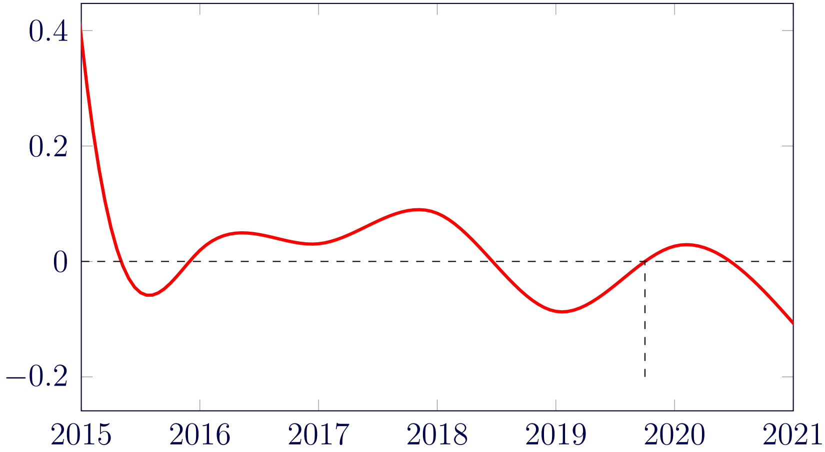
7 Normalising

- Calculate $c_{2019.75} = \sum_{j \geq 1} \hat{\kappa}_{0,j} B_j(2019.75)$.
- Re-balance with:
  - $\sum_{j \geq 0} (\hat{\kappa}_{0,j} - c_{2019.75}) B_j(y)$,
  - $\alpha' = \alpha + c_{2019.75}$, and
  - $\omega' = \omega + c_{2019.75}$.

…and the model fit is unchanged.

7 Combining $\hat{\kappa}_{0,j}$

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y) \text{ for } y \in [2015, 2021]$$

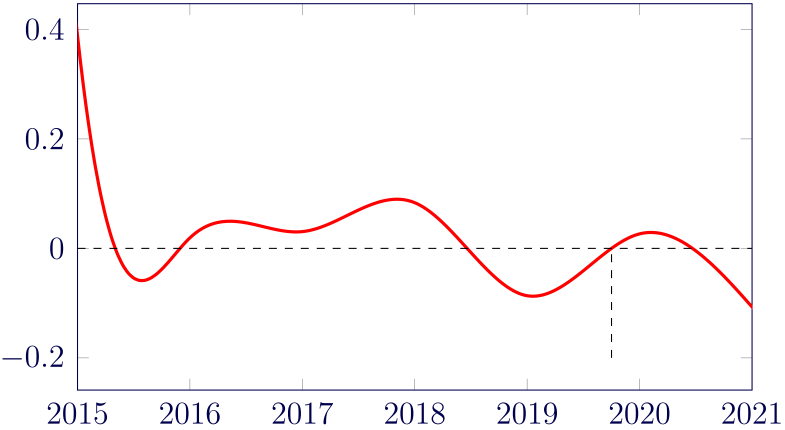


8 Seasons and shocks

- Previous slides used one-year knot spacing.
- What if we use half-year knot spacing?
- Or quarter-year knot spacing?

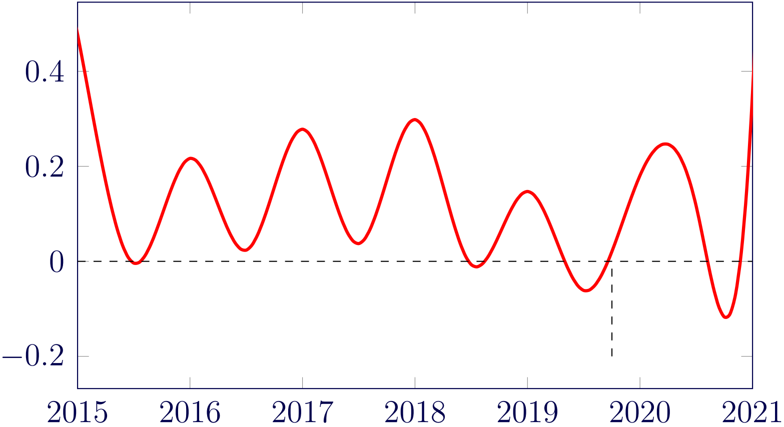
8 UK3, one knot per year

$$\sum_{j \geq 0} (\hat{\kappa}_{0,j} - c_{2019.75}) B_j(y)$$



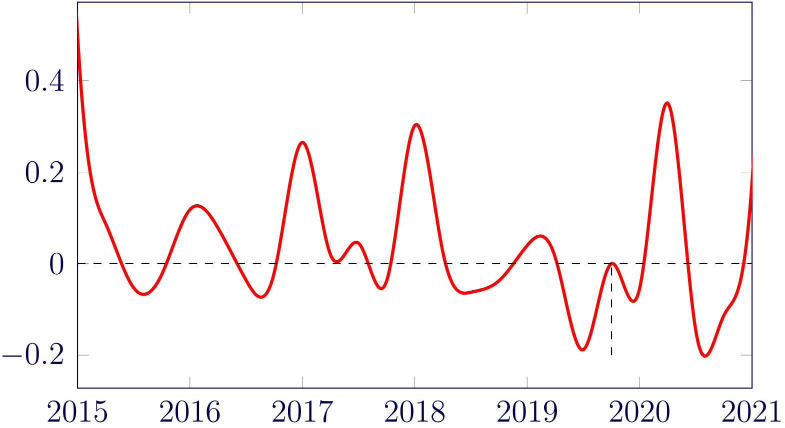
8 UK3, two knots per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



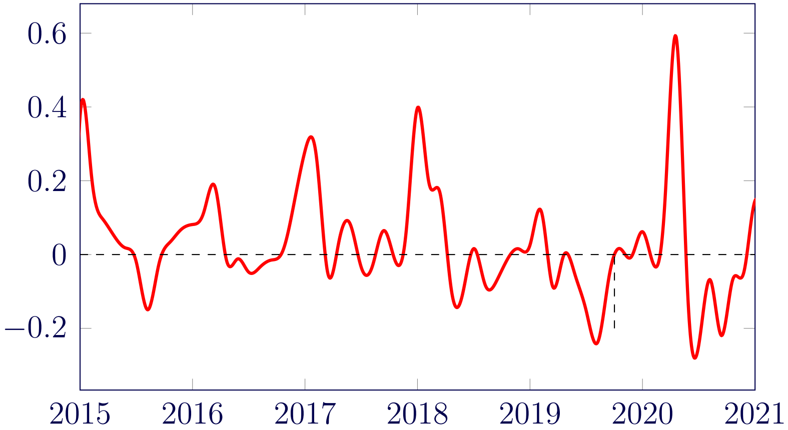
8 UK3, four knots per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



8 UK3, ten knots per year

$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



8 Seasons and shocks

- Half-year knot spacing reveals seasonal variation.
- 4 and 10 knots per year reveal covid-19 shock…

  ….but also introduce random variation pre-shock.

8 Measuring fit

Knots per year	Parameter count	AIC	BIC
1	14	187,594	187,729
2	20	187,412	187,605
4	32	187,324	187,634
10	68	187,244	187,901

Source: Richards [2021, Table 4].

8 Contradiction!

- AIC lowest with 10 knots per year.
- BIC lowest with 2 knots per year.
- AIC under-penalises parameters. . .

  . . .and leads to over-parameterisation.

8 Contradiction!

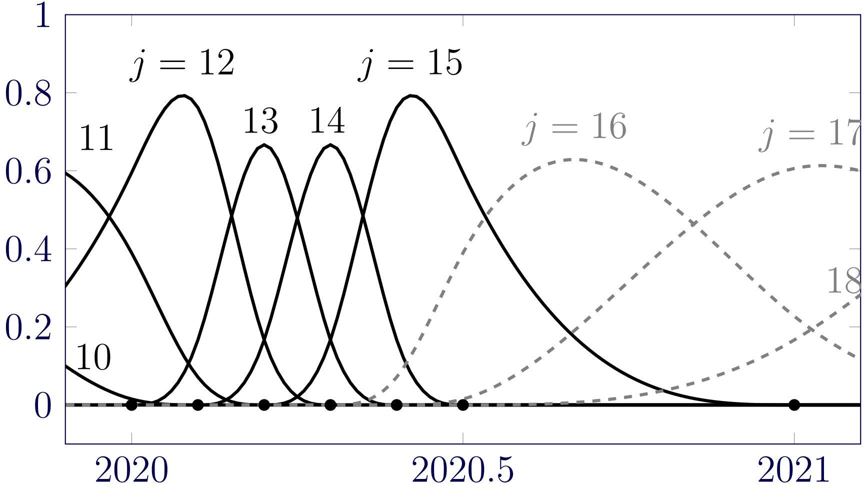
- This is not about the small-sample correction to the AIC [Hurvich and Tsai, 1989] ($n = 116,056$, so sample is not small!)
- Nor is this about a large parameter-to-observation ratio.
- Issue appears to be about number of degrees of freedom used when many parameters are insignificant; see discussion in Richards [2021, Section 12].

8 Variable knot spacing

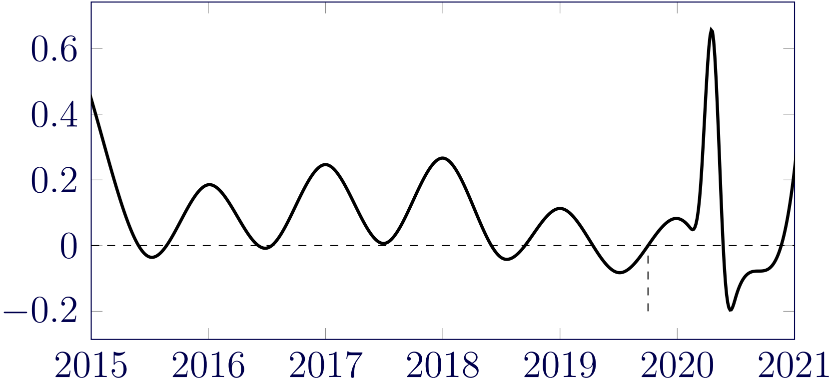
- Knots don't have to be equally spaced [Kaishev et al., 2016].
- Use two knots per year for seasonal variation…

  …and add knots where we know the shocks are.

Part of a basis of nineteen variably-spaced cubic B-splines.

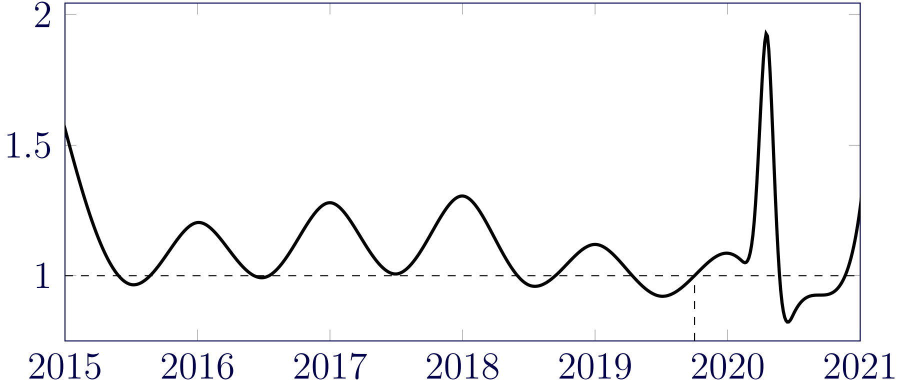


$$\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)$$



8 Variable knot spacing

$$\exp\left(\sum_{j\geq 0}(\hat{\kappa}_{0,j} - c_{2019.75})B_j(y)\right)$$



8 Variable knot spacing

- Seasonal variation means peak winter mortality is 15–30% higher than summer mortality.
- Mortality hazard doubled in April-May 2020 relative to baseline of October 2019.

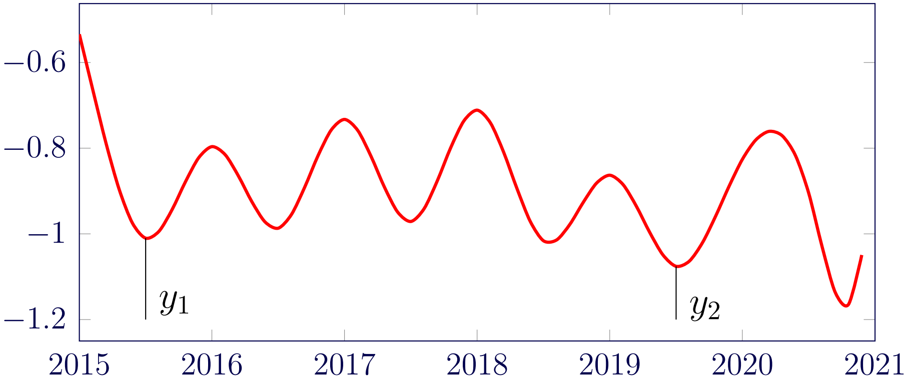
9 Mortality improvements

9 Mortality improvements

- We can also estimate portfolio-specific mortality improvements.
- Consider time component at y_1 v. y_2.
- Use midsummer points for stability.

9 UK3, two knots per year

$$\sum_{j \geq 1} \hat{\kappa}_{0,j} B_j(y) \text{ for } y \in [2015, 2021].$$



Annual improvement rate, i, between y_1 and y_2:

$$i_{y_1,y_2} = \left[1 - \exp\left(\frac{\sum_{j\geq 1} \hat{\kappa}_{0,j}\left[B_j(y_2) - B_j(y_1)\right]}{y_2 - y_1}\right)\right] \times 100\%$$

Source: Richards [2021].

9 Mortality improvement

- For UK3 aggregate annual improvement rate between mid-2015 and mid-2019 was 1.2% p.a.
- Can compare with CMI model used for reserving.

10 Conclusions

10 Conclusions — I

Modelling by age needs little flexibility

Use Hermite splines.

Modelling in time needs lots of flexibility

Use Schoenberg [1964] splines.

10 Conclusions — II

- Add knots around pandemic shocks.
- BIC better than AIC for model selection.
- Exercise judgement as to normal mortality level.
- Can estimate portfolio-specific improvement rate.

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Coronavirus graphic  from CDC

More on longevity risk at ▸ www.longevitas.co.uk

11 Legal matters

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