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Portfolio-specific mortality models

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Overview

1. About the speaker
2. Risk factors
3. Fitting multi-factor models
4. Communication
5. Measuring uncertainty
6. Conclusions

1 About the speaker

1 About the speaker

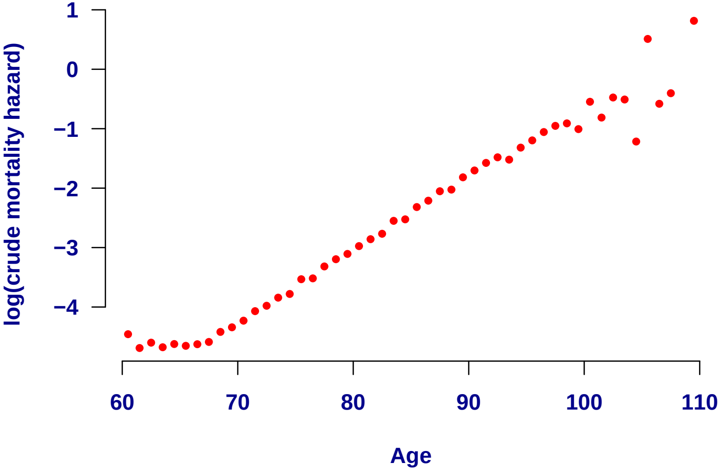
- Independent consultant on longevity risk since 2005.
- Founded longevity-related software businesses in 2006:

- Joint development with Heriot-Watt University in 2009:

2 Risk factors

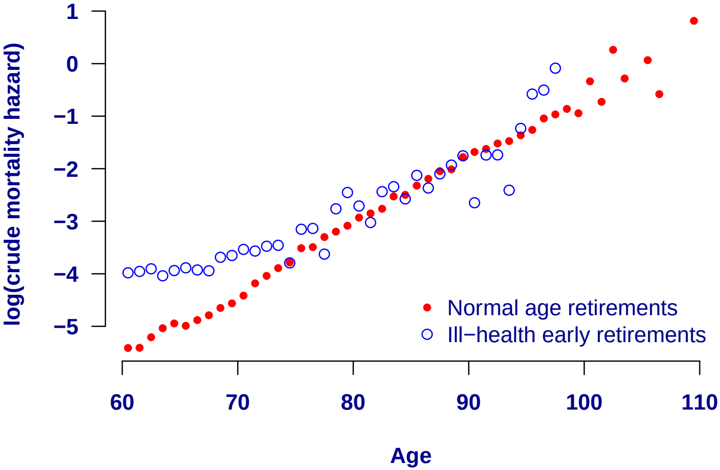
2 Risk factors

Mortality by age. Richards et al. [2013].



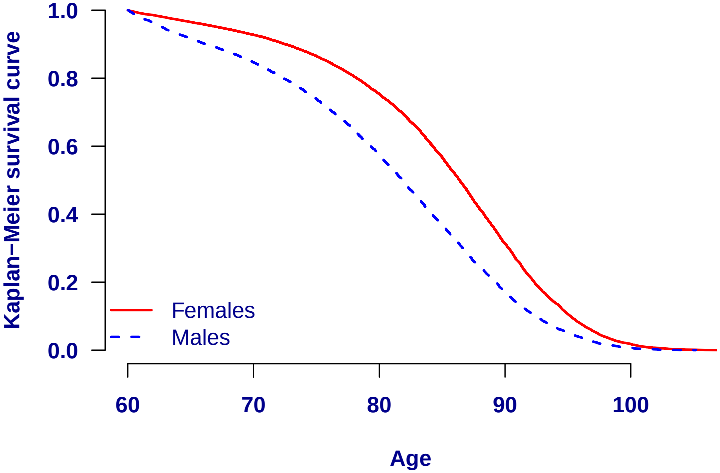
2 Risk factors

Mortality by health at retirement. Richards et al. [2013].

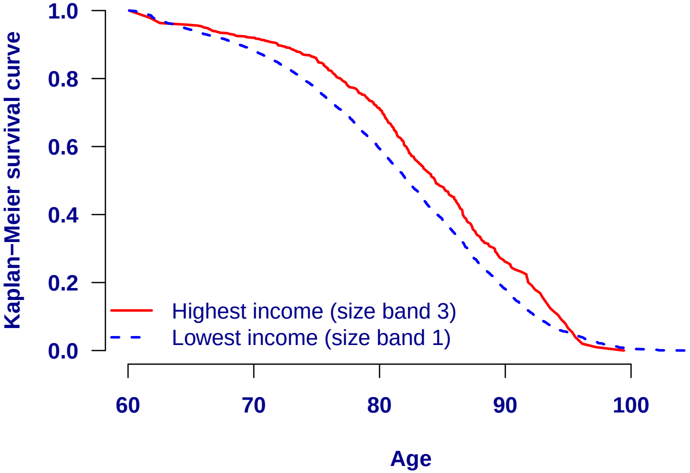


2 Risk factors

Survival curves for males and females. Richards et al. [2013].



Survival curves by pension size (males only). Richards et al. [2013].



2 Impact of risk factors I

UK annuitants from Richards and Jones [2004]:

Risk factor	**Change**	**Annuity factor**	**Relative change**
Base case	-	13.39	
Gender	Female→male	12.14	-9.3%
Lifestyle	Top→bottom	10.94	-9.9%
Duration	Short→long	9.88	-9.7%
Pension size	Largest→smallest	9.36	-5.2%
Region	South→North	8.90	-4.9%
Overall			-33.6%

2 Impact of risk factors II

German pensioners from Richards et al. [2013]:

Risk factor	Change	Annuity factor	Relative change
Base case	-	16.11	
Gender	Female→male	14.53	-9.8%
Retirement health	Normal→ill-health	12.97	-10.7%
Pension size	Largest→smallest	11.72	-9.7%
Region	B→P	11.02	-5.9%
Employer type	Private→public	10.60	-3.9%
Overall			-34.2%

2 Impact of risk factors

- Different portfolios have different risk factors available.
- Important to use risk factors relevant to *your* business processes.

2 Risk factors: lifestyle

Q. What was "Lifestyle" for the UK annuitants?

A. Profile based on the annuitant's address or postcode. . .

2 Digression: UK postcodes

- UK has a hierarchical postcode structure.
- Each piece of postcode narrows in on a geographical area.
- Hierarchical postcodes in UK, USA, Canada and the Netherlands.

2 Digression: UK postcodes

Anatomy of a UK postcode

2 Digression: UK postcodes

- Compare the postcodes EH4 4SP and EH3 6BX.
- Both in Edinburgh.
- Life expectancy "1.1 years less than the UK average"[1]

[1] Punter Southall, Postcode Life Expectancy Tool, accessed on 5th May 2015.

2 Digression: UK postcodes

EH4 4SP. Source: Google Maps, accessed 5th May 2015.

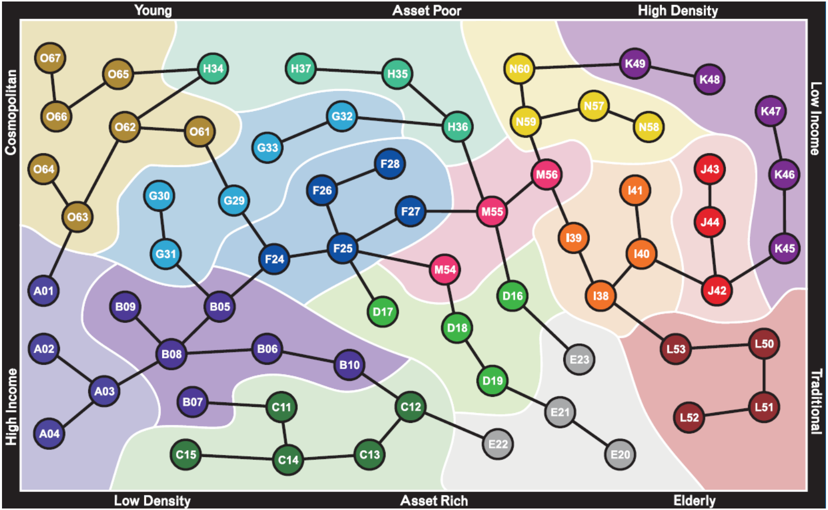
2 Digression: UK postcodes

EH3 6BX. Source: Google Maps, accessed 5th May 2015.

2 Digression: UK postcodes

- There are around 1.7 million residential postcodes.
- We can't use Google Maps every time ☺
- Solution is to map each postcode to a *geodemographic type code*...

Mosaic family tree



2 Digression: UK postcodes

- EH4 4SP → K46 Municipal Challenge, High-Rise Residents.
- EH3 6BX → A01 City Prosperity, World-Class Wealth.
- 1.7 million residential postcodes become 67 lifestyle codes.

2 Geodemographic profiling

Works in:

- UK.
- USA.
- Canada.
- The Netherlands.

Does not appear to work in France.

Would it work in Sweden?

3 Fitting multi-factor models

3 Fitting multi-factor models

Q. Why not partition the data set and model deaths as Poisson counts?

A. Low cell counts limit the number of risk factors we can use…

Deaths per risk-factor combination (Richards et al. [2013]).

Member of largest scheme	Region	Scheme type	Pension size-band	Normal retirees: Females	Males	Ill-health retirees: Females	Males	Widow(er)s: Females	Males
No	B	1	1	5,142	5,313	525	738	4,434	618
			2	824	725	39	98	36	0
			3	282	413	14	33	24	1
		2	1	2,200	1,323	308	183	628	222
			2	305	275	20	39	18	0
			3	140	206	15	18	15	1
	P	1	1	695	811	51	99	798	89
			2	138	122	7	22	9	0
			3	59	72	1	5	3	1
		2	1	174	274	26	33	166	23
			2	26	56	3	4	4	0
			3	8	41	5	2	5	0
Yes	B	1	1	480	338	41	45	224	47
			2	108	65	12	3	4	0
			3	60	45	1	3	4	0
Totals				10,641	10,079	1,068	1,325	6,372	1,002

3 Fitting multi-factor models

- Previous slide has over 30,000 deaths.
- However, 21 cells have fewer than five deaths. . .
- . . .and we still haven't partitioned by age.

3 Fitting multi-factor models

- This approach is called *stratification*.
- It severely limits the number of risk factors we can use.

3 Fitting multi-factor models

- It is better to model mortality at the level of the individual.
- Individual-level models have no limits on the number of risk factors.
- Models for the force of mortality, μ_x, offer the greatest flexibility.

3 Fitting multi-factor models

Gompertz [1825] model:

$$\mu_x = e^{\alpha + \beta x} \tag{1}$$

3 Fitting multi-factor models

Gompertz [1825] model for each individual:

$$\mu_{x_i} = e^{\alpha_i + \beta_i x_i} \tag{2}$$

3 Fitting multi-factor models

$$\alpha_i = \alpha_0 + \sum_j \alpha_j z_{i,j} \tag{3}$$

$$\beta_i = \beta_0 + \sum_j \beta_j z_{i,j} \tag{4}$$

where

$$z_{i,j} = \begin{cases} 1 & \text{if life } i \text{ has risk factor } j, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

3 Fitting multi-factor models

Wide choice of mortality laws:

$$\text{Makeham [1859]:} \qquad \mu_x = e^{\epsilon} + e^{\alpha+\beta x} \tag{6}$$

$$\text{Perks [1932]:} \qquad \mu_x = \frac{e^{\alpha+\beta x}}{1 + e^{\alpha+\beta x}} \tag{7}$$

$$\text{Beard [1959]:} \qquad \mu_x = \frac{e^{\alpha+\beta x}}{1 + e^{\alpha+\rho+\beta x}} \tag{8}$$

More choices in Richards [2008] and Richards [2012].

3 Fitting multi-factor models

Find parameter estimates by maximising log-likelihood:

$$\ell = \sum_i H_{x_i}(t_i) + \sum_i d_i \log \mu_{x_i} \tag{9}$$

where

$$H_{x_i}(t_i) = \int_0^{t_i} \mu_{x_i+s} ds \tag{10}$$

and

$$d_i = \begin{cases} 1 & \text{if life dead at age } x_i + t_i, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

4 Communication

4 Communication

- Communicating a multi-factor model is tricky.
- Model might be too commercially valuable to disclose in detail.
- Conversion to a reference table solves both issues...

4 Communication

- Equate reserves under private bespoke basis (B) and public reference basis (T).
- For pensions or annuities this means solving:

$$\sum_i w_i \bar{a}_{x_i}^T = \sum_i w_i \bar{a}_{x_i}^B \qquad (12)$$

- Usually solve separately for males and females.

4 Communication

Model in Richards et al. [2013] has seven risk factors besides age and time:

- Gender.
- Normal v. ill-health retirement.
- Pension size.
- Region.
- Scheme/employer type.
- First life v. surviving spouse.
- Membership of largest city scheme.

Equivalent percentages of Sterbetafel Deutschland 2009–2011.

Member of largest scheme	Region	Scheme type	Pension size-band	Normal retirees: Females	Males	Ill-health retirees: Females	Males	Widow(er)s: Females	Males
No	B	1	1	90.9	95.7	129.0	148.9	105.2	114.9
			2	86.6	84.5	120.1	123.2	98.7	98.5
			3	77.9	75.8	107.4	109.2	88.4	87.9
		2	1	84.4	88.3	118.5	135.5	97.2	105.4
			2	81.5	79.2	112.1	114.3	92.5	91.9
			3	73.4	71.1	100.4	101.5	83.0	82.1
	P	1	1	99.5	108.6	145.4	175.6	116.6	132.3
			2	93.6	93.6	132.3	140.0	107.5	110.2
			3	83.8	83.5	117.6	123.4	95.9	97.8
		2	1	92.0	99.7	132.8	159.0	107.3	120.9
			2	87.7	87.3	123.0	129.4	100.4	102.5
			3	78.6	78.0	109.5	114.2	89.7	91.1
Yes	B	1	1	82.6	86.2	115.5	131.7	95.0	102.8
			2	80.0	77.6	109.8	111.8	90.8	90.0
			3	72.1	69.7	98.4	99.3	81.5	80.4

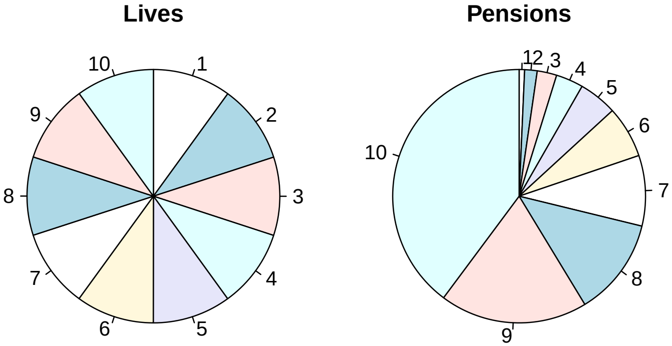
5 Measuring uncertainty

5 Measuring uncertainty

- Consider UK pension scheme in Richards [2014].
- 17,067 benefit records, of which 2,265 were deaths.
- A model for age, gender, pension size and time trend is fitted.
- Equivalent-reserve calculation equates to 88.5%/87.2% of S2PA mortality table.
- What uncertainty surrounds these numbers?
- What is a 1:200 stress for mis-estimation risk?

5 Concentration of risk

Pension scheme data from Richards [2014, Appendix I].



- Top tenth of pensioner population receives 39.8% of all pensions.
- Next two tenths of pensioner population receive further 31.4%.

5 Concentration of risk

Parameter estimates for model with age, gender, pension size and time. From Richards [2014, Table 6].

Parameter	Estimate	Standard error	Contributing lives
Age	0.148	0.005	15,698
Gender.M	0.479	0.060	5,956
Intercept	-14.731	0.491	15,698
Makeham	-5.420	0.154	15,698
Pension size — medium	-0.180	0.078	3,140
Pension size — largest	**-0.313**	**0.108**	**1,567**
Time	-0.046	0.016	15,698

Lives with largest pensions have lowest mortality, but estimate also has greater uncertainty.

5 Measuring uncertainty

Q. How do we measure the financial impact of this uncertainty?

A. Use the covariance matrix for the parameter estimates…

5 Measuring uncertainty

Basic procedure:

- Fit a parametric statistical model to portfolio's experience data.
- Use the covariance matrix to generate alternative parameter sets.
- Value in-force liabilities using the alternative parameter sets.
- Collect liability valuations into set, S.

5 Measuring uncertainty

- S is a sample of the distribution of financial impact of mis-estimation.
- S can then be analysed to understand mis-estimation risk. . .

5 Quantifying mis-estimation risk

Let S_p be the p^{th} quantile of S. Then:

- $S_{0.5}$ is the median or central liability.
- $S_{0.025}$ and $S_{0.975}$ give a 95% confidence interval for the liability.
- $S_{0.995}$ is the 99.5% Solvency II stressed liability.

In practice we quote the mis-estimation capital as:

$$\left(\frac{S_{0.995}}{S_{0.5}} - 1\right) \times 100\% \qquad (13)$$

Can use mean of S in place of $S_{0.5}$ (difference is usually negligible).

5 Impact of size of data set

99.5% mis-estimation capital as percentage of best-estimate reserve:

Data set	Date range	Number of lives	Mis-estimation capital
UK pensioners	2007–2012	15,698	4.4–4.7%
German pensioners	2007–2011	244,908	1.1–1.2%

→ Larger portfolios with more data need less mis-estimation capital.

6 Conclusions

6 Conclusions

- Use risk factors relevant to your business.
- Use survival models to create portfolio-specific tables. . .
- . . . and for measuring mis-estimation risk.
- Equate results to reference table for communication.

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