Heriot-Watt University, Edinburgh

P-spline smoothing in actuarial work

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Contents

1. About the speaker

1. About the speaker

- Graduated twice from Heriot-Watt: 1990 (BSc) and 2012 (PhD).
- Consultant on longevity risk since 2005.
- Founded longevity-related analytics businesses in 2006:

LONGEVITAS

mortalityrating.com

- Joint venture with Heriot-Watt in 2009:

PROJECTIONS TOOLKIT

2. Actuarial exceptionalism

2. Actuarial exceptionalism

Actuaries use their own jargon:

Actuarial term	**Statistical term**
central exposed-to-risk	total waiting time
force of mortality	mortality hazard
graduation	smoothing

3. Graduating mortality tables

3. Graduating mortality tables

- Actuaries use mortality tables for premium calculations and reserving.
- These tables are created from experience data.
- Actuaries use the term "graduation" for smoothing mortality rates...
- ... and historically did this directly and without a statistical model.

3. Graduating mortality tables

- Using a statistical model is preferable:

$$D_x \sim \mathrm{Poisson}(E_x^c \mu_x)$$

- D_x is the number of deaths at age x last birthday.
- E_x^c is the central exposed-to-risk in $[x, x+1)$.
- μ_x is the mortality hazard.

3. Graduating mortality tables

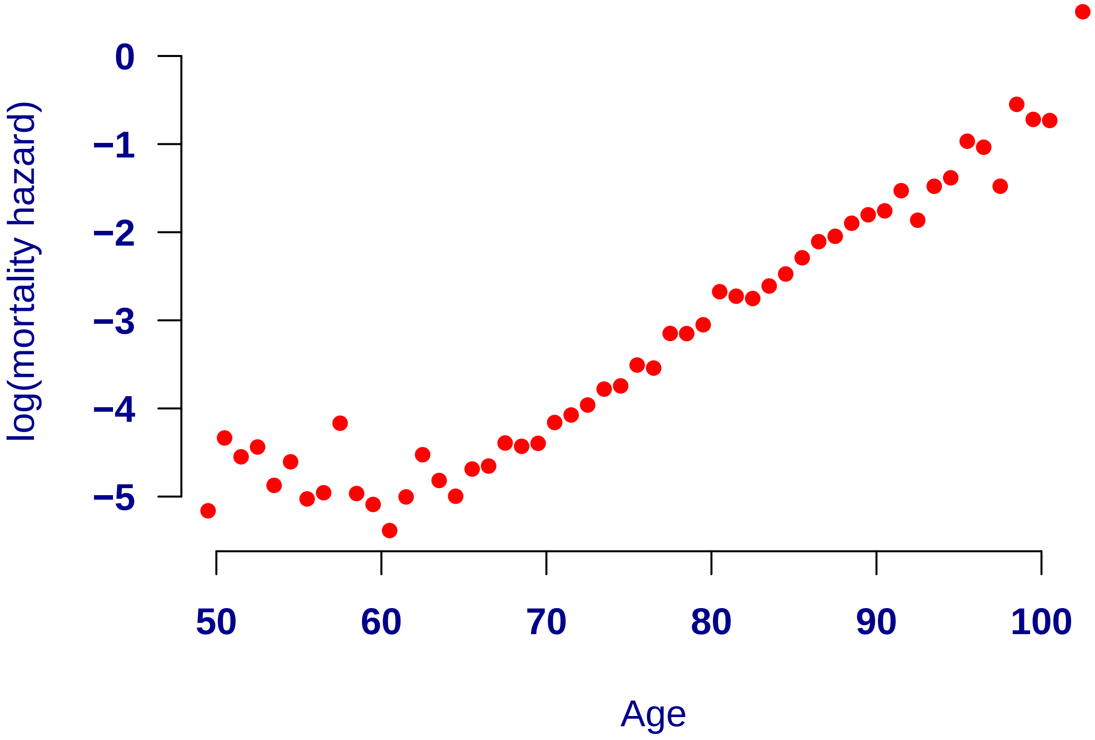
Smoothing of μ_x is indirect via penalised splines:

$$\mu_x = \sum_j^m B_j(x)\theta_j$$

- $B_j(x)$ is the j^{th} basis spline evaluated at x.
- θ_j is a parameter to be estimated.

3. Graduating mortality tables

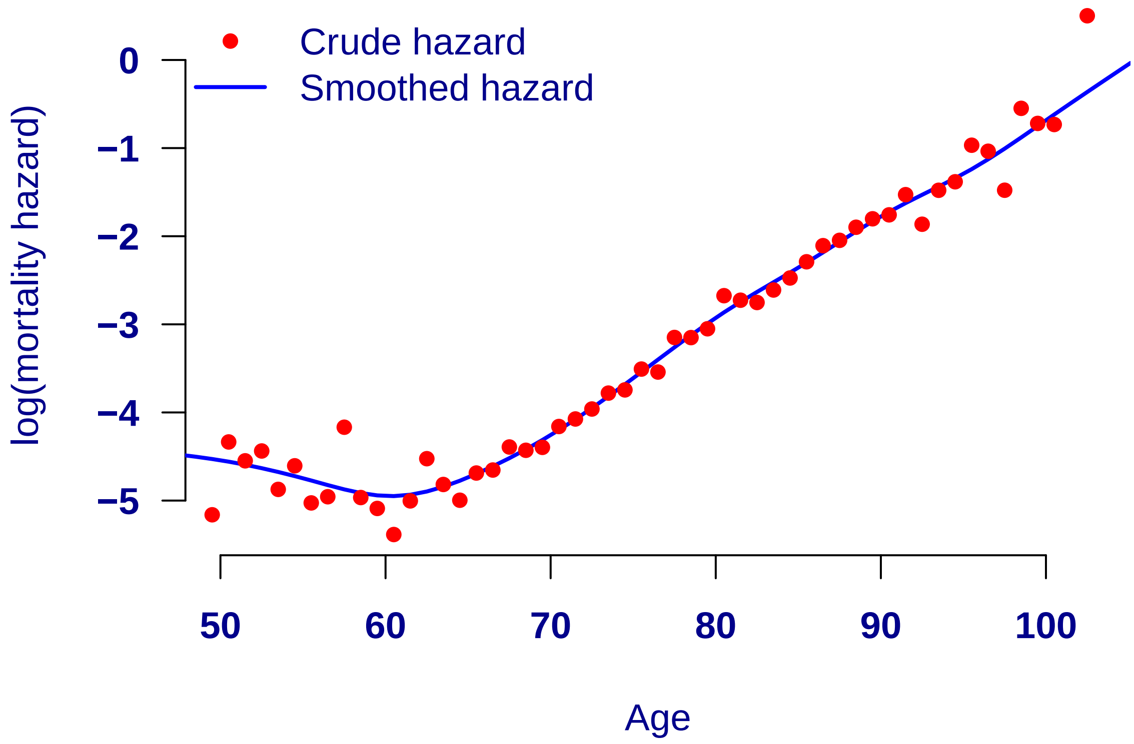
Problem 1 — crude mortality rates contain random fluctations:



Source: Mortality data for 2007–2012 for medium-sized pension scheme in England and Wales.

3. Graduating mortality tables

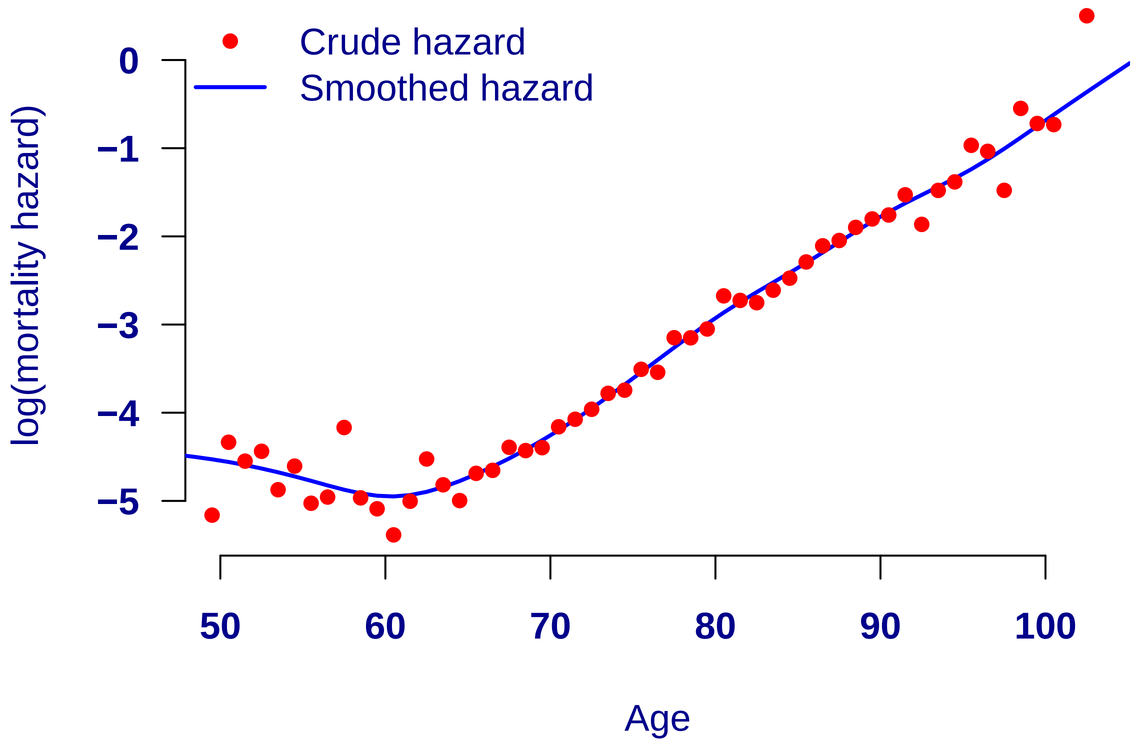
Solution 1 — smooth indirectly using penalised splines:



Source: Mortality data for 2007–2012 for medium-sized pension scheme in England and Wales. Software from Richards & Currie (2012).

3. Graduating mortality tables

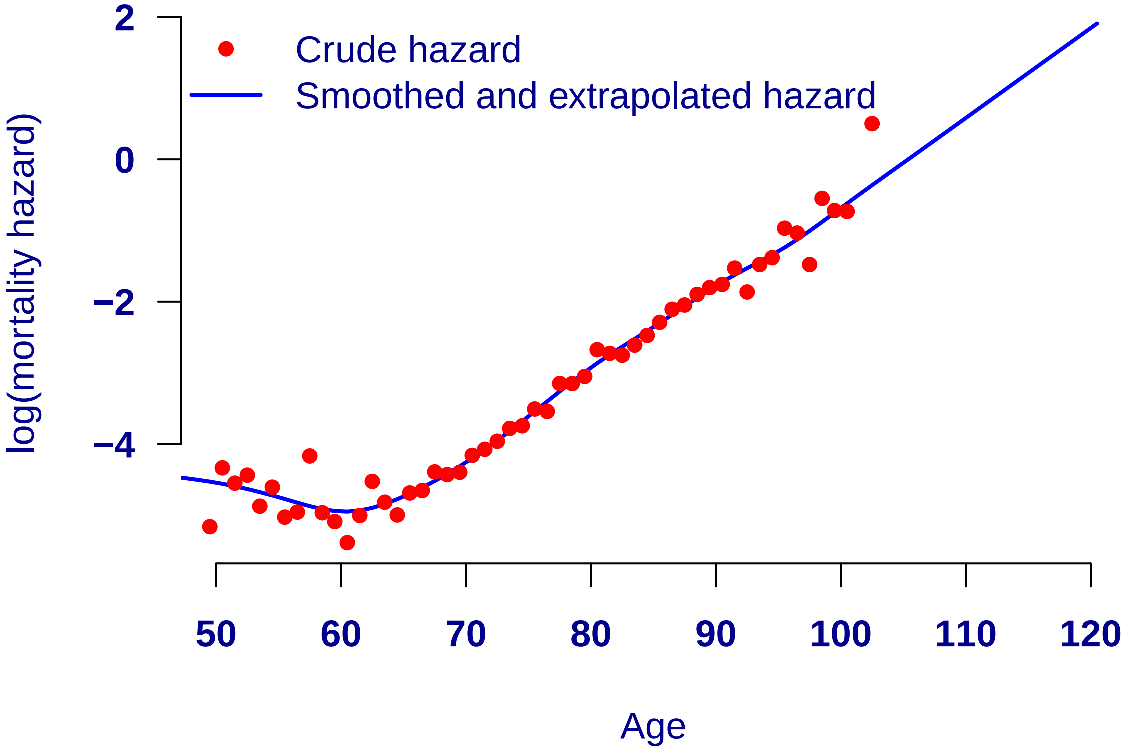
Problem 2 — no data above age 103, but need rates up to age 120:



Source: Mortality data for 2007–2012 for medium-sized pension scheme in England and Wales. Software from Richards & Currie (2012).

3. Graduating mortality tables

Solution 2 — use penalty function to extrapolate to higher ages:



Source: Mortality data for 2007–2012 for medium-sized pension scheme in England and Wales. Software from Richards & Currie (2012).

3. Graduating mortality tables

Penalised splines in a statistical model solve three problems at once:

1. The greatest weight is given to areas with the most data.
2. Smooth fitted rates are produced.
3. Automatic extrapolation to ages with no data.

4. Mortality forecasts

4. Mortality forecasts

- Actuaries need to assess uncertainty over future mortality trends.
- Reserves must cover, say, 99.5% of scenarios over coming year.
- Stochastic projection models will play a big role in Solvency II...

4. Mortality forecasts

Consider the model from Lee & Carter (1992):

$$\log \mu_{x,y} = \alpha_x + \beta_x \kappa_y$$

where x is age, y is calendar year.

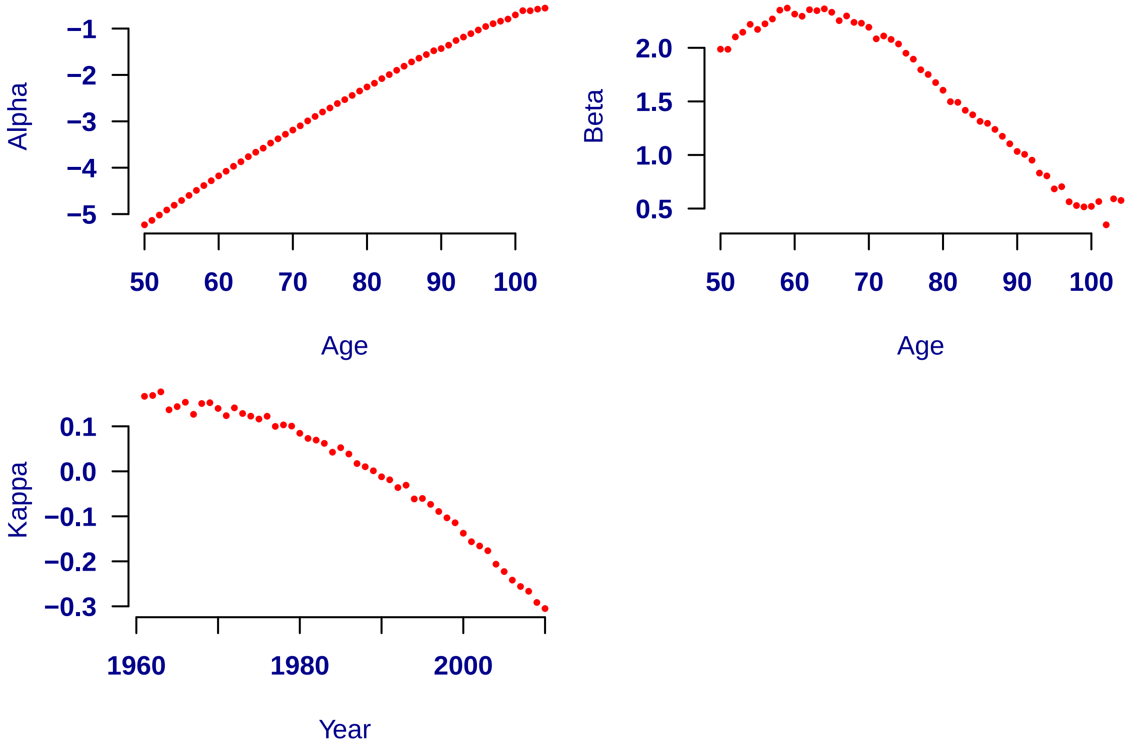
We impose the following identifiability constraints:

$$\sum_y \kappa_y = 0$$

$$\sum_y \kappa_y^2 = 1$$

4. Mortality forecasts

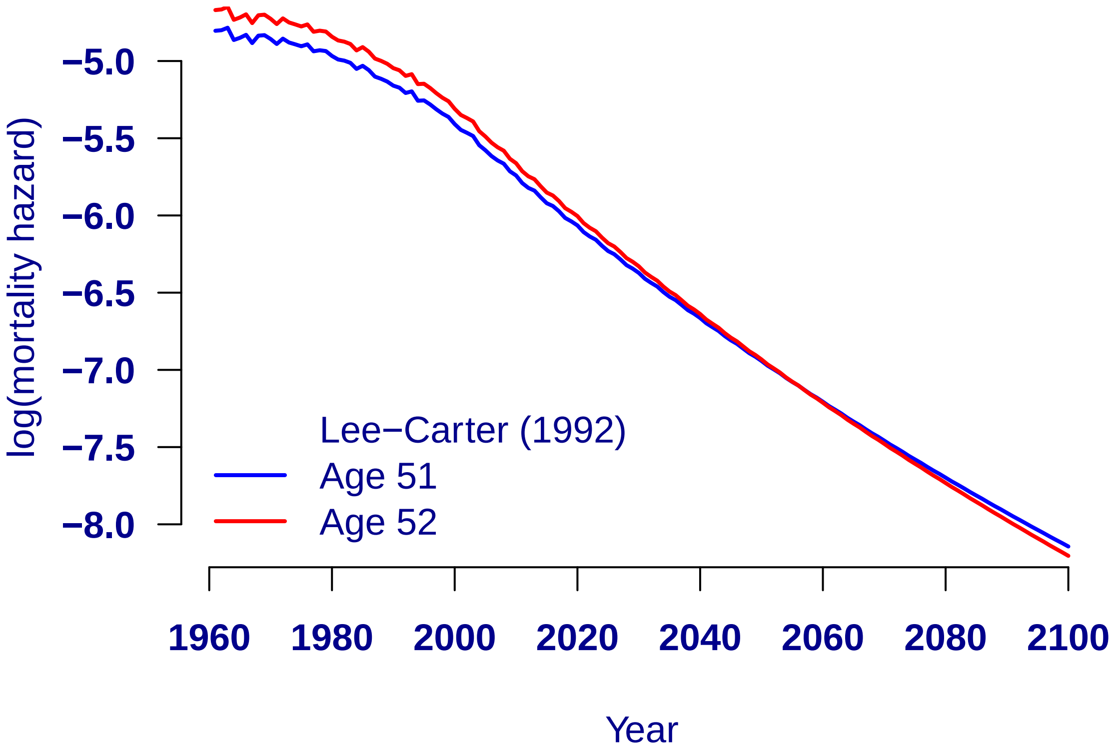
Problem 1 — model is over-parameterised:



Source: Mortality data for 1961–2012 for males ages 50–104 in the U.K.

4. Mortality forecasts

Problem 2 — inconsistent long-range forecasts from irregular β_x:



Source: Model from Lee & Carter (1992) fitted to mortality data for 1961–2012 for males ages 50–104 in the U.K.

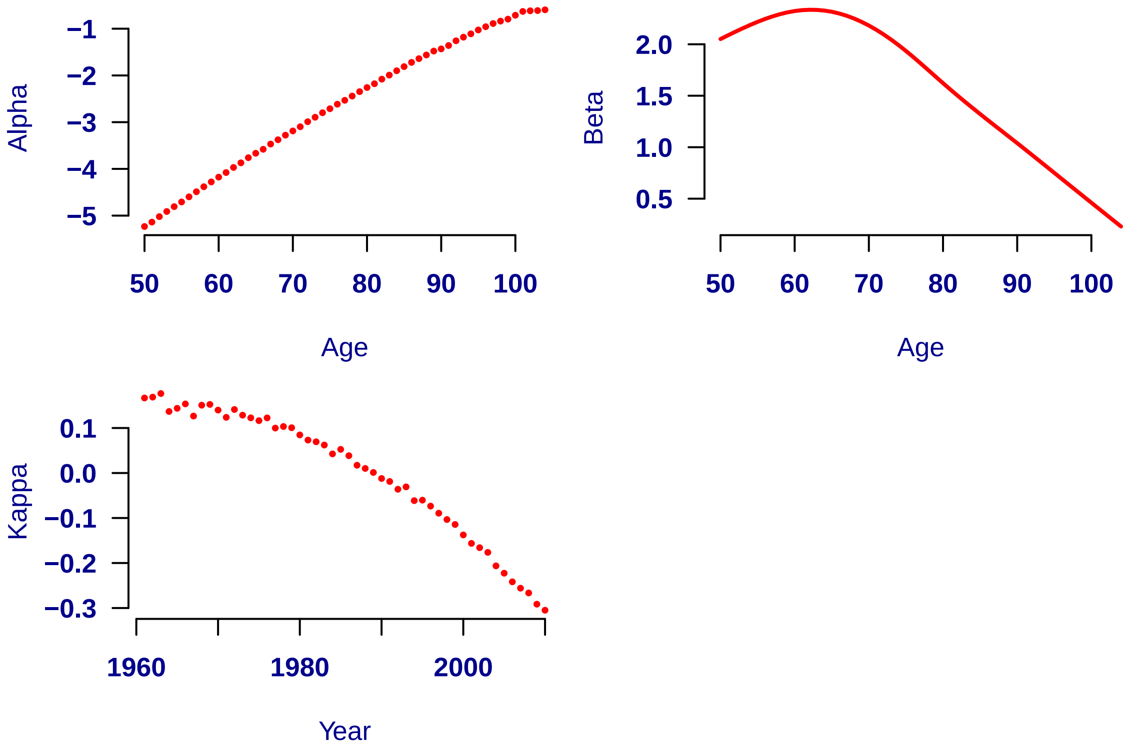
4. Mortality forecasts

Solution 1 — use splines to reduce effective number of parameters.

Solution 2 — use P-spline smoothing to produce smooth β_x.

This is the model from Delwarde, Denuit & Eilers (2007)...

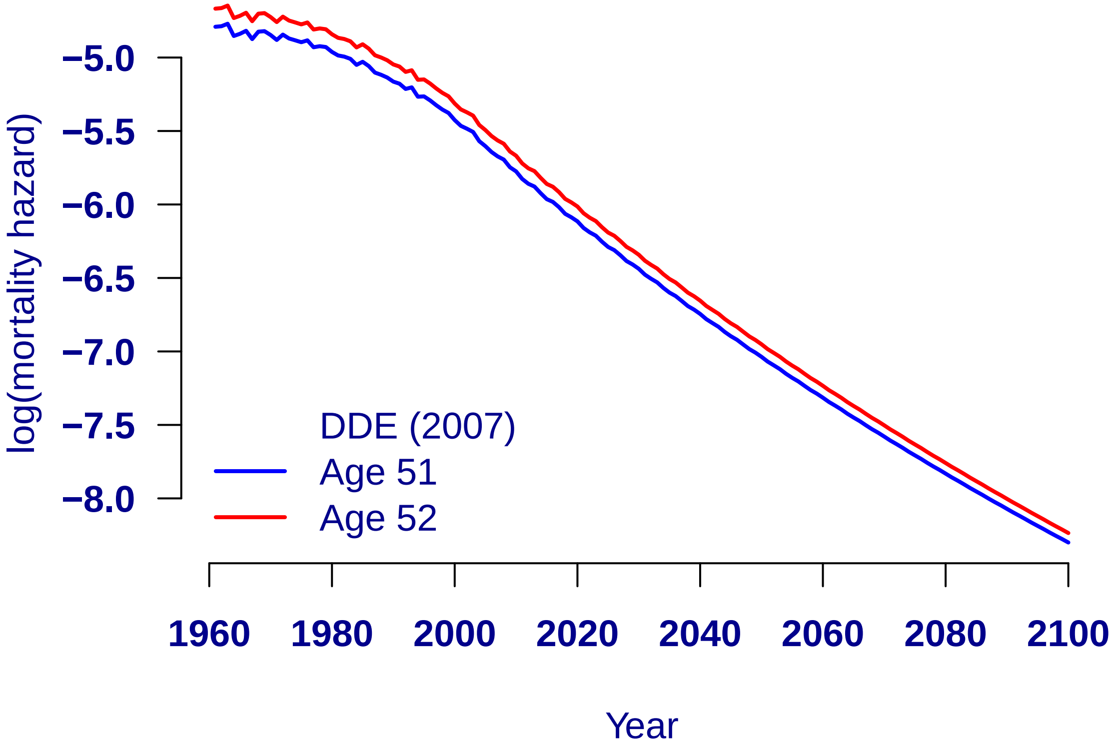
4. Mortality forecasts



Source: Mortality data for 1961–2012 for males ages 50–104 in the U.K.

4. Mortality forecasts

DDE model produces consistent long-range forecasts at adjacent ages:



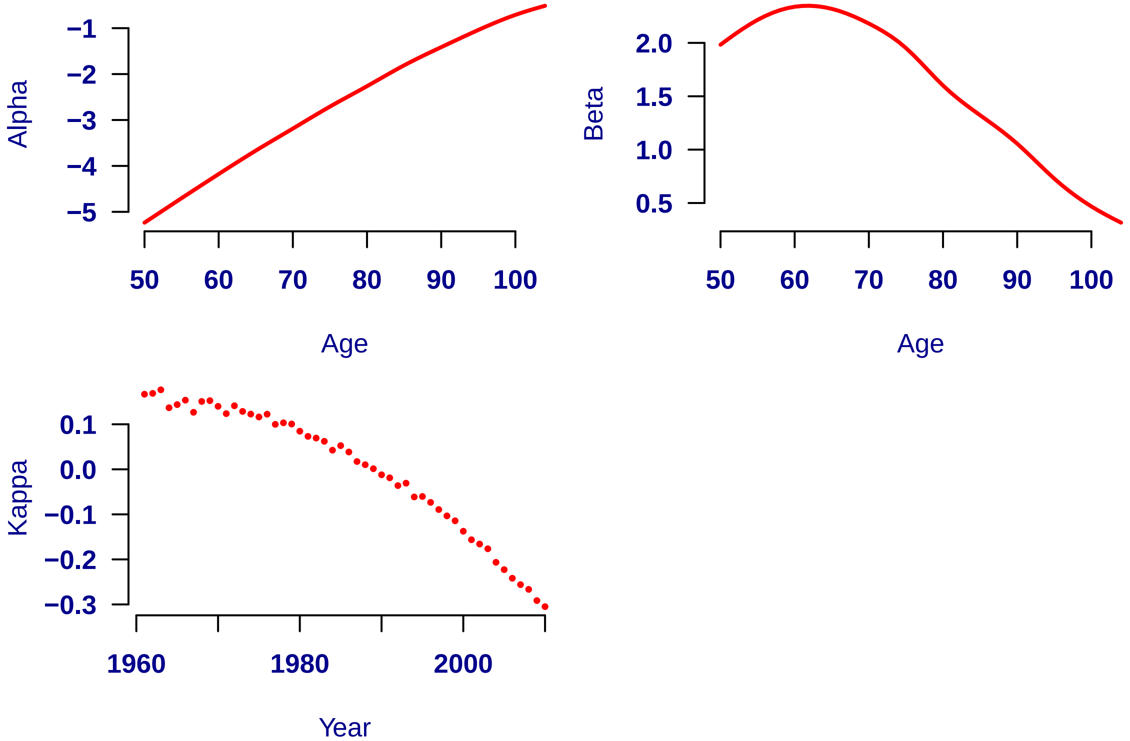
Source: Model from Delwarde, Denuit & Eilers (2007) fitted to mortality data for 1961–2012 for males ages 50–104 in the U.K.

4. Mortality forecasts

- Why stop at smoothing β_x?
- Could smooth α_x as well.

This is the LC(S) model from Currie (2013)...

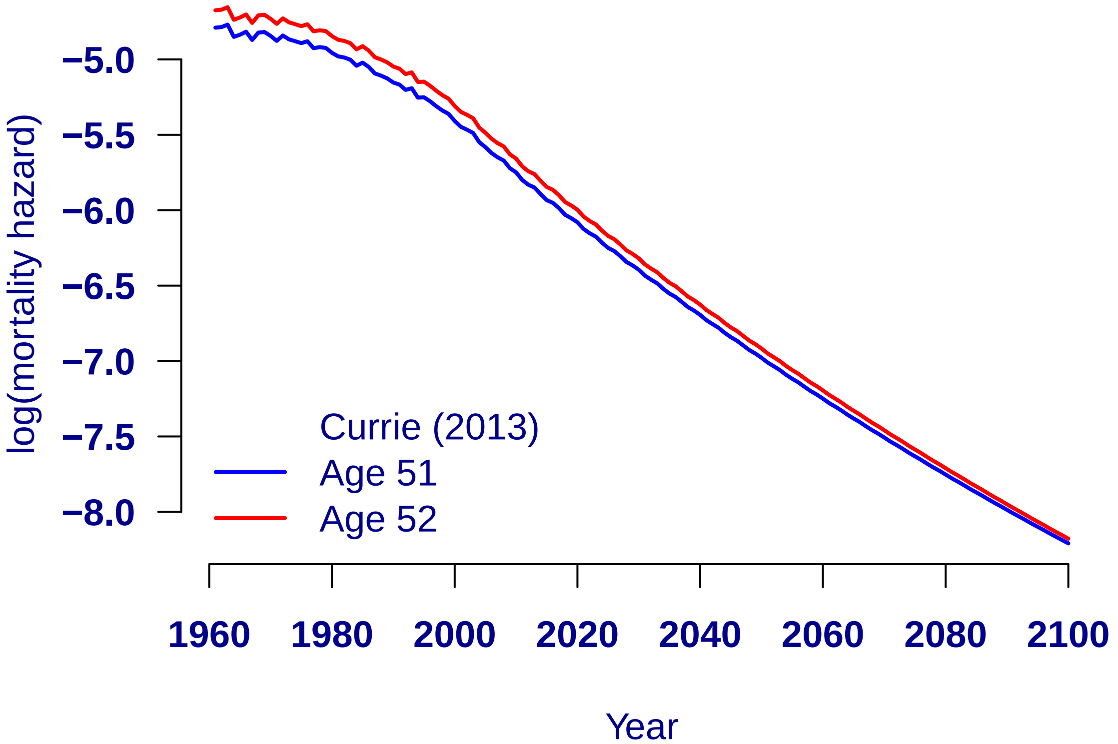
4. Mortality forecasts



Source: Mortality data for 1961–2012 for males ages 50–104 in the U.K.

4. Mortality forecasts

LC(S) model also avoids crossover at adjacent ages:



Source: Model from Currie (2013) fitted to mortality data for 1961–2012 for males ages 50–104 in the U.K.

4. Mortality forecasts

Q. Why not smooth κ_y as well?

A. We can — see Richards & Currie (2009), but this takes us into topics such as over-dispersion. . .and beyond the scope of this presentation!

4. Mortality forecasts

P-splines in mortality projection models:

1. Reduce the effective dimension in over-parameterised models.
2. Help produce more consistent long-range forecasts.

5. Conclusions

5. Conclusions

P-splines are useful in actuarial work for:

- Graduating mortality tables.
- Extrapolating mortality tables.
- Reducing over-parameterisation.
- Ensuring consistent mortality forecasts.

References

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