Hotel Park Inn, Amsterdam

The many uses of VaR for longevity trend risk

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Overview

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2. Some questions
3. Trend risk v. one-year view
4. Trend risk v. multi-year view
5. Deferred annuities
6. VaR v. CTE
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8. Conclusions

1 About Longevitas

1 Longevitas Ltd

- Founded 2006.
- Based in Edinburgh.
- Clients in UK, USA, Canada and Switzerland.
- Research partnership with Heriot-Watt.

1 Services for actuaries

- Experience analysis and mis-estimation:

  LONGEVITAS

- Stochastic mortality projections and capital:

  PROJECTIONS TOOLKIT

- Rating pension schemes:

  mortalityrating.com

2 Some questions

2 Some questions:

- How do you put a multi-year trend risk into a one-year view?
- How do different product types behave?
- How do VaR and CTE regimes compare?
- How do you value an index-based hedge?

2 Translation table

VaR	Value-at-Risk
CTE	Conditional Tail Expectation
LC	Model from Lee and Carter [1992]
APC	Age-Period-Cohort model
M5/CBD	Model from Cairns et al. [2006]
M6	Model from Cairns et al. [2009]
(S)	Smoothing as per Eilers and Marx [1996]
2DAC	Model from Richards et al. [2006]

3 Trend risk v. one-year view

3 Trend risk v. one-year view

"Whereas a catastrophe can occur in an instant, longevity risk takes decades to unfold"

The Economist [2012]

3 Trend risk v. one-year view

- Longevity trend risk unfolds over many years.
- Insurance regulations have a one-year view of risk.
- How do you reconcile the two?

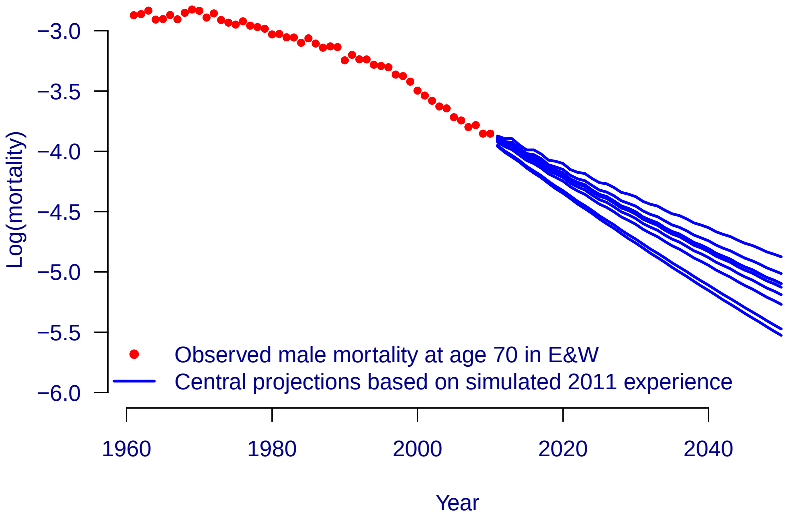
3 Trend risk v. one-year view

Solution from Richards et al. [2014]:

1. Simulate next year's experience data.
2. Refit the projection model.
3. Value liabilities.
4. Discard simulated experience data.

Repeat (1)–(4) a few thousand times...

3 Sensitivity of forecast



Source: Lee-Carter example from Richards et al. [2014].

3 Liability values

- Our unknown liability is X (say).
- VaR-style solvency capital:

$$\left(\frac{Q_\alpha}{\mathbb{E}[X]} - 1\right) * 100\%$$

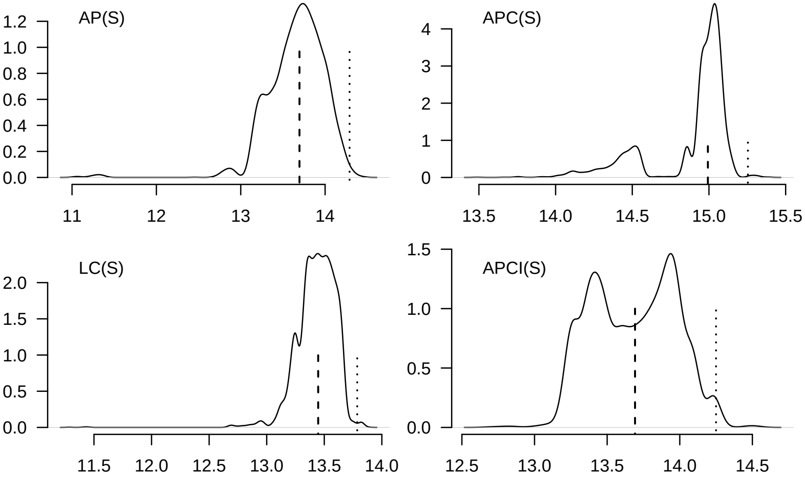
where Q_α is α-quantile of X, i.e. $\Pr(X < Q_\alpha) = \alpha$.

3 Simulated liability values

- We don't know the distribution of X...

  ... but we do have a sample of simulations.
- Estimate $\mathbb{E}[X]$ from mean of sample.
- Estimate Q_α from sample using Harrell and Davis [1982].

3 One-year liability densities



Annuities payable to male aged 70. Means marked with dashed line and $Q_{99.5\%}$ marked with dotted line. Source: Richards et al. [2017, Table 4].

3 Value-at-risk

- Variety of density shapes.

  $\Rightarrow$ not all unimodal

  ... and not all symmetric.

- Considerable variability between models.

  $\Rightarrow$ need to use multiple models

  ... and exercise *actuarial judgement*.

4 Trend risk v. multi-year view

4 Multi-year view

- Richards et al. [2014] was for one-year insurer solvency.
- The same methodology has other applications...

4 Multi-year view

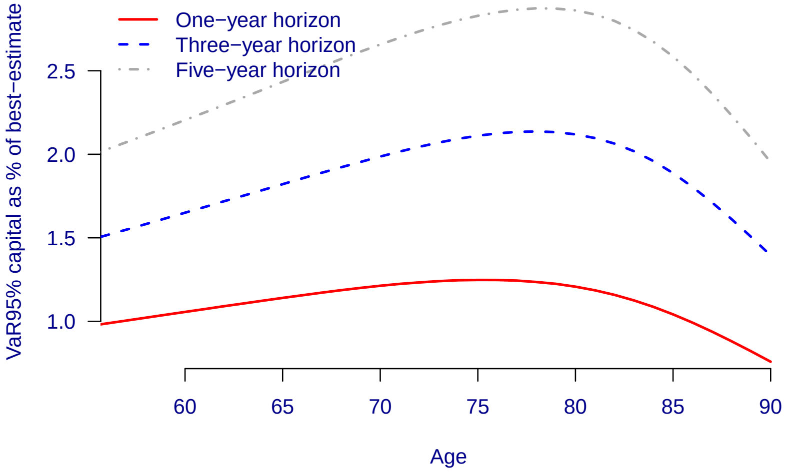
Medium-term business planning:

- 3–5 years for insurer ORSA.
- Ten-year "glide path" to buy-out for pension schemes.

4 Multi-year view

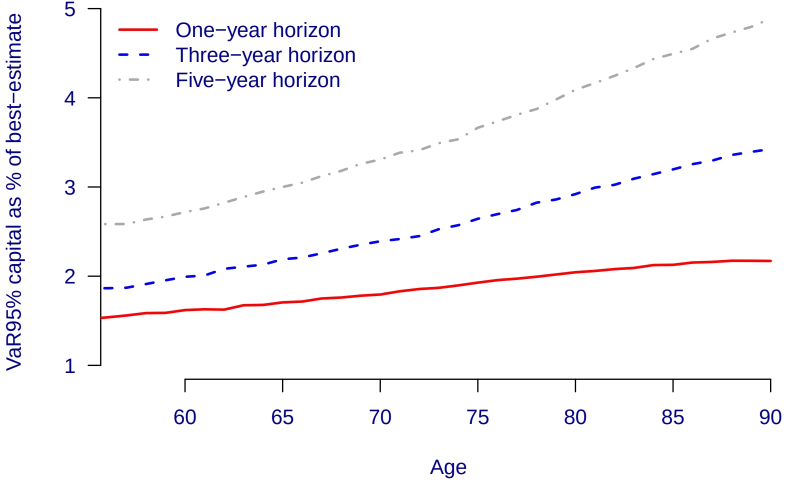
- Take one-year framework from Richards et al. [2014].
- Extend time horizon to 3–5 years.
- Reduce p-value to, say, 95%...

4 Females, LC(S)



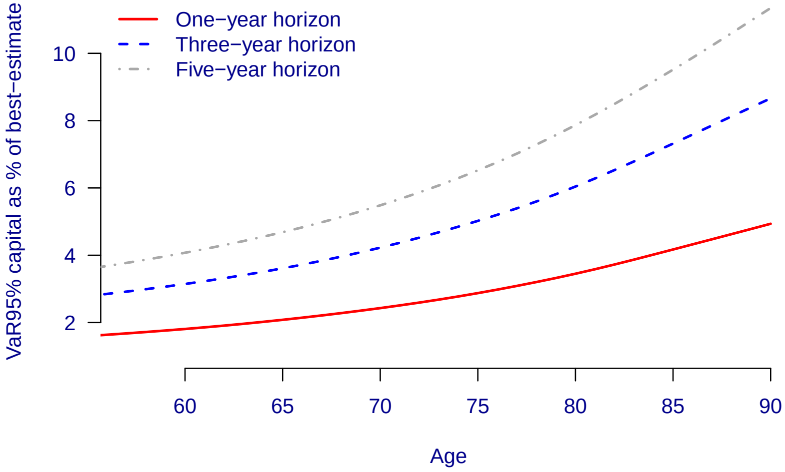
Immediate annuities under Lee-Carter model. UK data ages 50–104, 1971–2016

4 Females, APC(S)



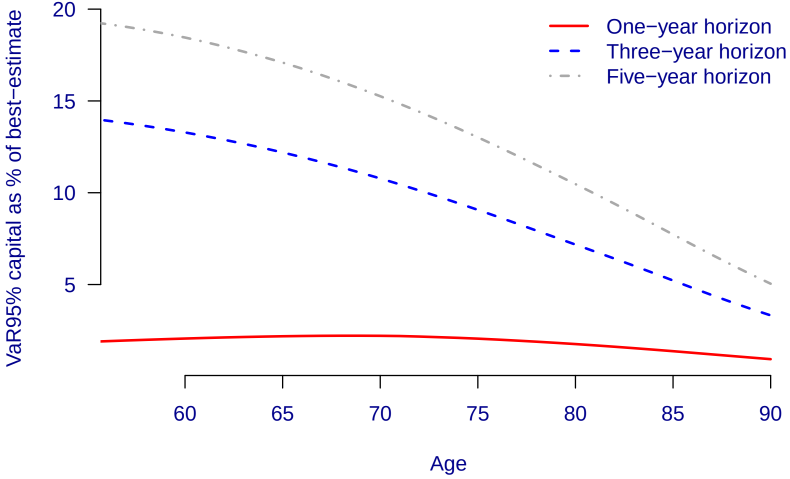
Immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016

4 Females, M5(S)



Immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016

4 Males, LC(S)



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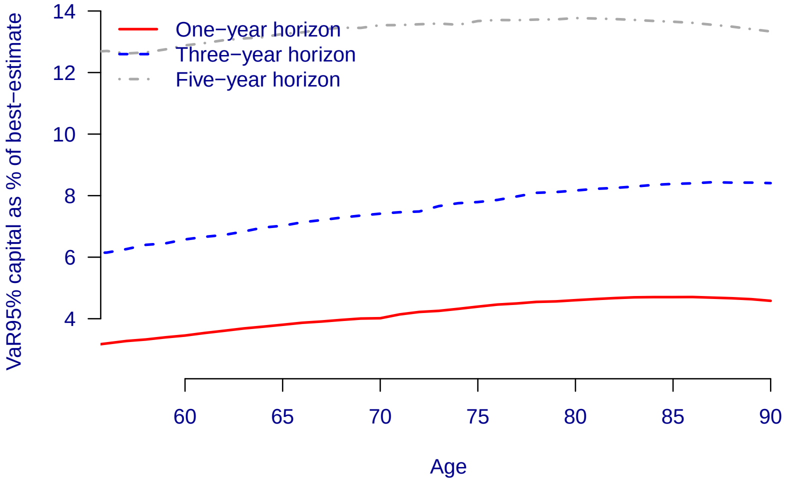

Immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016

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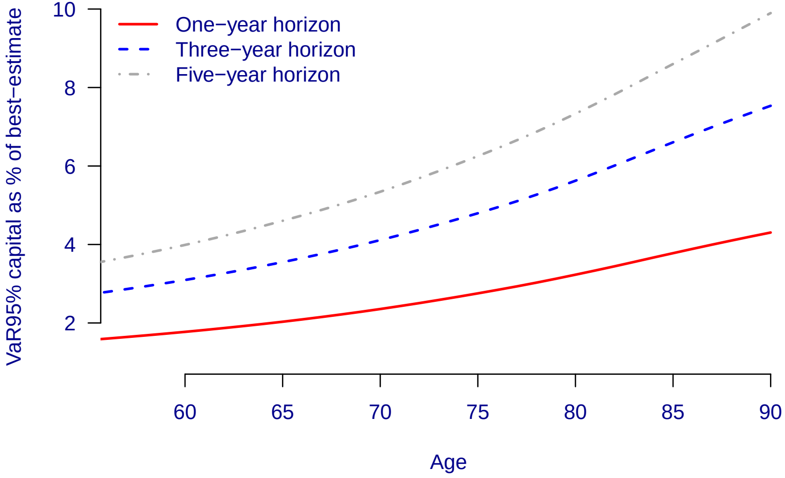


Immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016

4 Multi-year view

- No consistent pattern in capital by term.
- Considerable variability between models.

$\Rightarrow$ need to use multiple models

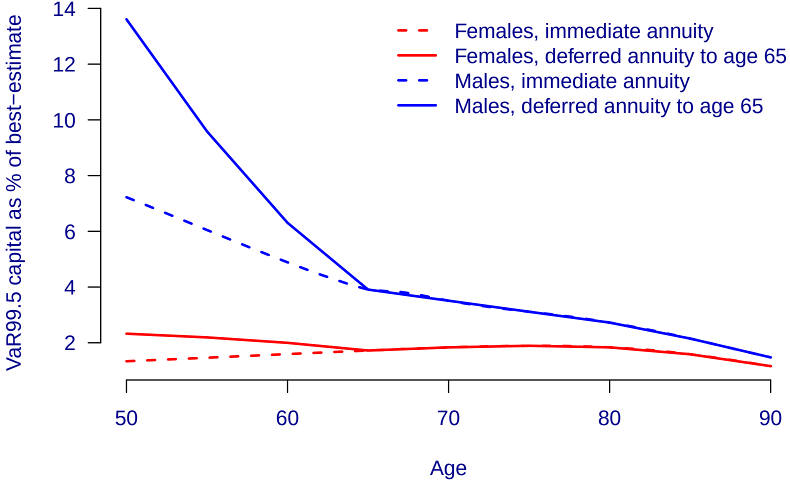
... and exercise *actuarial judgement* (again).

5 Deferred annuities

5 Deferred annuities

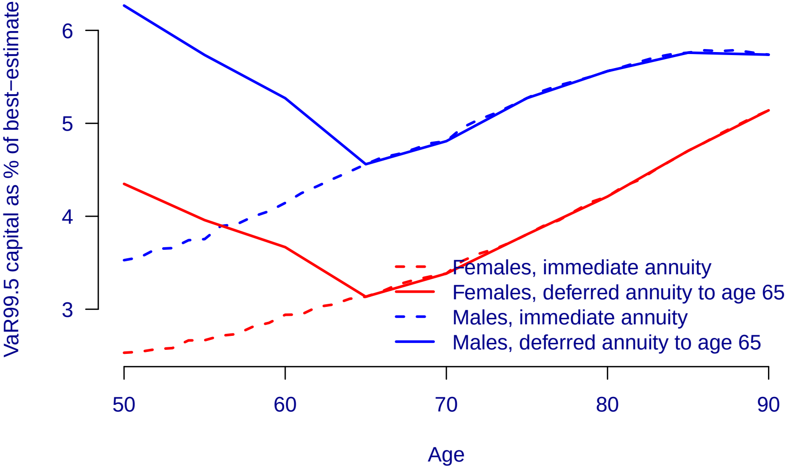
- Most published work concerns immediate annuities and pensions in payment.
- What about deferred annuities and pensions?
- Assume payment from age 65.
- Compare VaR99.5% solvency capital for immediate and deferred annuities.

5 Solvency capital, LC(S)



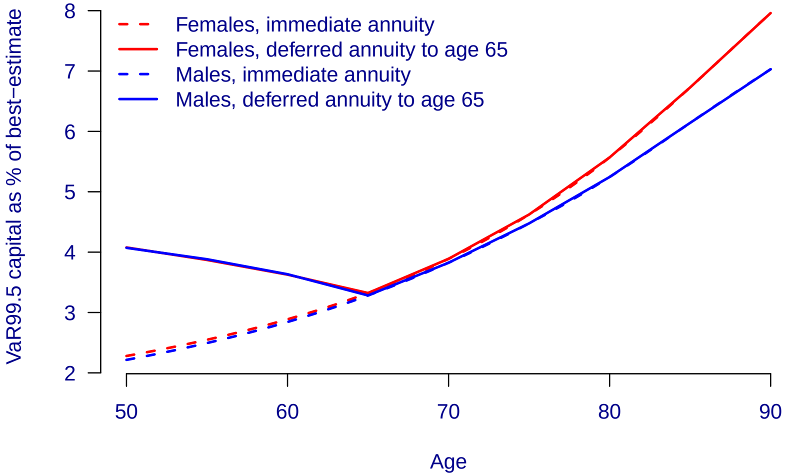
Deferred and immediate annuities under Lee-Carter model. UK data ages 50–104, 1971–2016

5 Solvency capital, APC(S)



Deferred and immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016

5 Solvency capital, CBD (M5)



Deferred and immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016

5 Deferred annuities

- Depending on age, solvency capital for deferred annuities can be double that of annuities in payment.
- Sharp differences in solvency capital by gender.

6 VaR v. CTE

- Our unknown liability is X (say).
- VaR-style solvency capital:

$$\left(\frac{Q_\alpha}{\mathbb{E}[X]} - 1\right) * 100\%$$

where Q_α is α-quantile of X, i.e. $\Pr(X < Q_\alpha) = \alpha$.

6 CTE

LONGEVITAS

- Our unknown liability is X (say).
- CTE-style solvency capital:

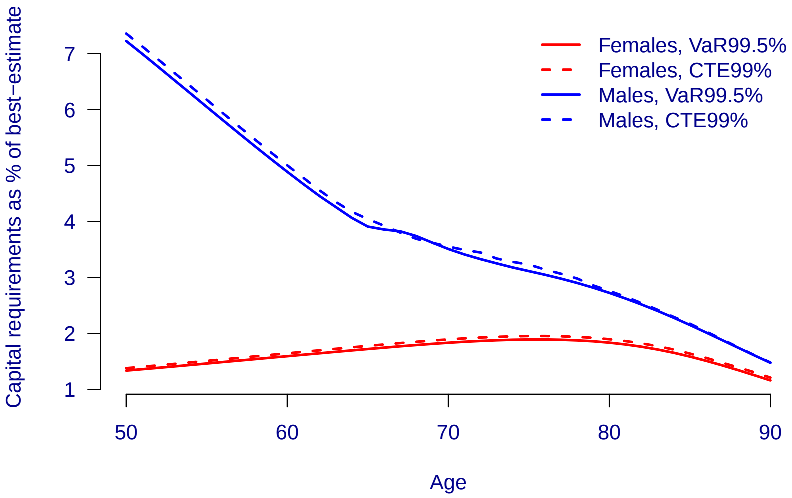
$$\left(\frac{\mathbb{E}[X|X \geq Q_\alpha]}{\mathbb{E}[X]} - 1\right) * 100\%$$

where Q_α is α-quantile of X, i.e. $\Pr(X < Q_\alpha) = \alpha$.

6 VaR v. CTE

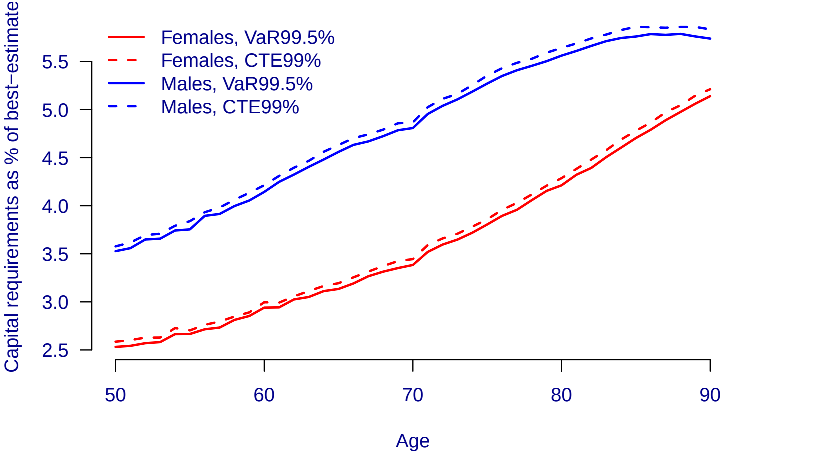
- How does VaR capital compare to CTE capital?
- $CTE_\alpha > VaR_\alpha$ (obviously!)
- But how does VaR99.5% compare to CTE99%?
- Can calculate both from same sample…

6 VaR v. CTE — LC(S) model



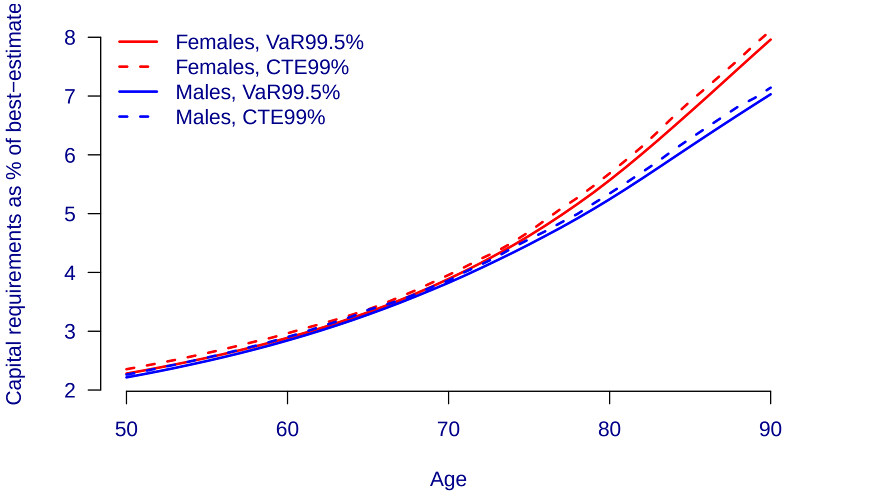
Annuities in payment under Lee-Carter model. UK data ages 50–104, 1971–2016

6 VaR v. CTE — APC(S) model



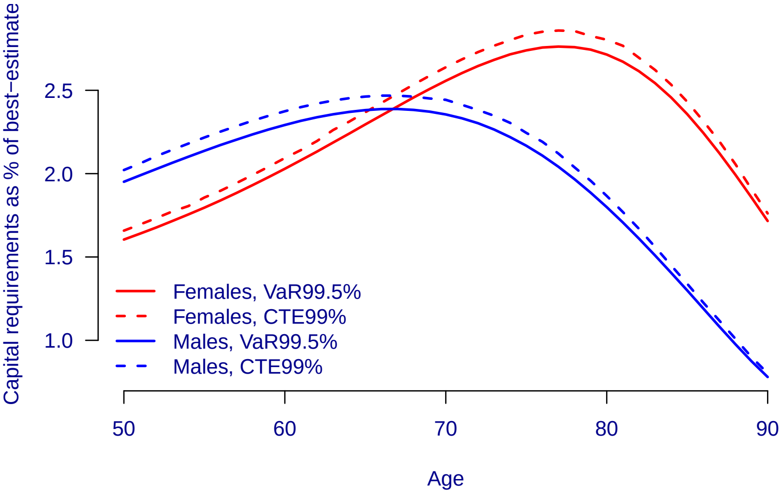
Annuities in payment under APC(S) model. UK data ages 50–104, 1971–2016

6 VaR v. CTE — M5 model



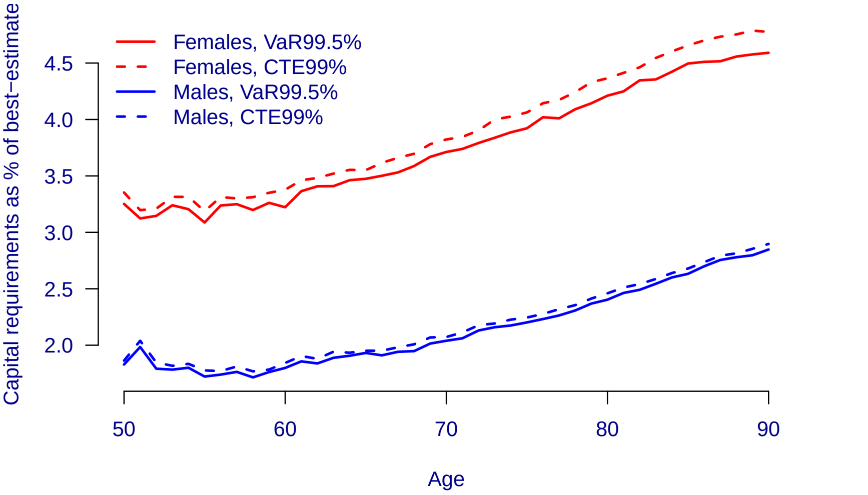
Annuities in payment under M5(S) model. UK data ages 50–104, 1971–2016

6 VaR v. CTE — LC(S)



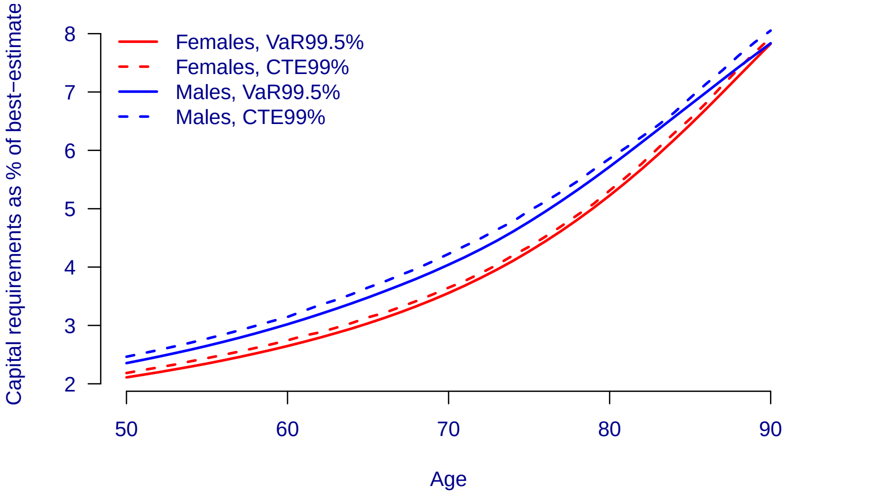
Annuities in payment under LC(S) model. UK data ages 50–104, 1971–2016

6 VaR v. CTE — APC(S)



Annuities in payment under APC(S) model. UK data ages 50–104, 1971–2016

6 VaR v. CTE — M5(S)



Annuities in payment under M5(S) model. UK data ages 50–104, 1971–2016

6 VaR v. CTE

- Longevity trend-risk capital very comparable between VaR99.5% and CTE99%.
- CTE99% usually slightly more prudent than VaR99.5%.
- Difference usually under 0.1%.

7 Index-based hedges

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- Population mortality (basis risk).
- Term n years.
- At end of term, fit Lee-Carter model (say) and use to value annuity with unknown value X.
- Use a function of X to close out the contract.

  $\Rightarrow$ This is just another multi-year VaR calculation.

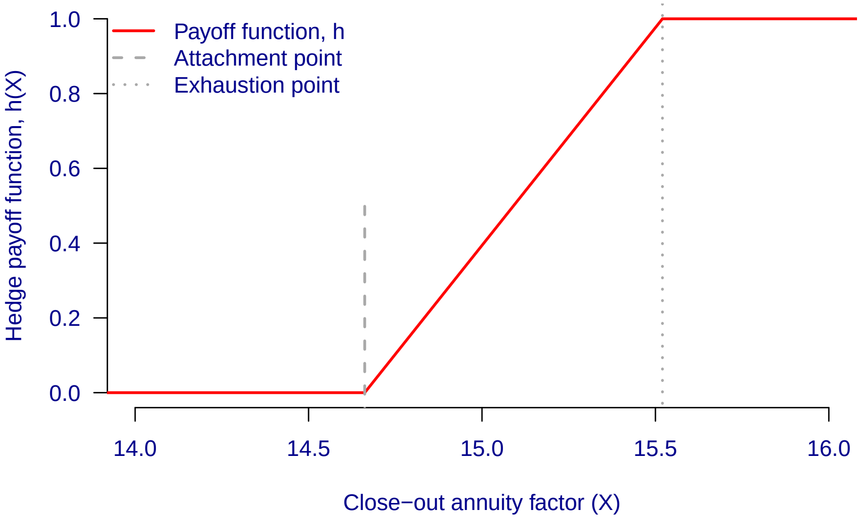
7 Index-based hedges

- Risk metric (annuity value) is X.
- Only pay above attachment point, AP.
- Pay no more than exhaustion point, EP.
- Standardise payoff, h, as:

$$h(X) = \max\left(0, \min\left(\frac{X - AP}{EP - AP}, 1\right)\right)$$

- See Cairns and El Boukfaoui [2017] for detailed discussion.

7 Hedge payoff function



7 Index-based hedges

- Set $AP = Q_{\alpha_1}$ and $EP = Q_{\alpha_2}$ $(\alpha_1 < \alpha_2)$.
- Q_α set with reference to Lee-Carter sample paths over n years, i.e. an n-year VaR simulation.
- Probability of payoff is $1 - \alpha_1$.
- Mean payoff can be estimated from VaR results.

7 Example hedge contract

- $n = 15$ years.
- Use Lee-Carter model for close-out calculation.
- Follow Cairns and El Boukfaoui [2017] and set $AP = Q_{60\%}$ and $EP = Q_{95\%}$.
- Probability of a payoff is 0.4.
- Average payoff is 0.375 (from 5,000 simulations).

7 Model risk

- Lee-Carter model used for both sample paths over n years **and** for payoff calculation.
- Assume we keep the Lee-Carter model for payoff calculation and also keep the same attachement and exhaustion points.
- What happens if the sample paths follow a *different* model?

7 Hedge assessment

Impact of different sample-path models on payoff:

Model	Payoff prob.	Mean payoff
LC(S)	0.40	0.375
M5(S)	0.53	0.592
2DAC	0.80	0.434
M6	0.82	0.710

Source: own calculations using population data for males in Netherlands, ages 50–104, 1971–2016. Annuity values discounted at 2% p.a.

7 Hedge assessment

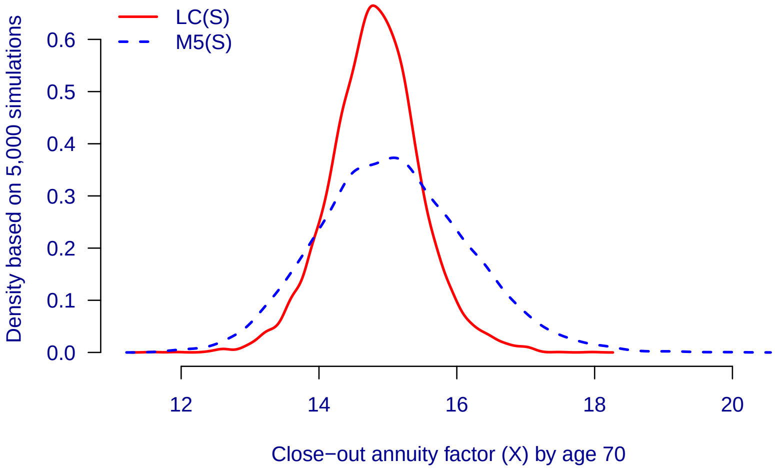
- Model for future mortality is unknowable (model risk).
- So payoff probability and expected payoff are also unknowable.
  - What value should the hedge contract have on the balance sheet?
  - What solvency capital relief should be given?

$\Rightarrow$ Actuarial judgement required on both counts.

7 Hedge assessment

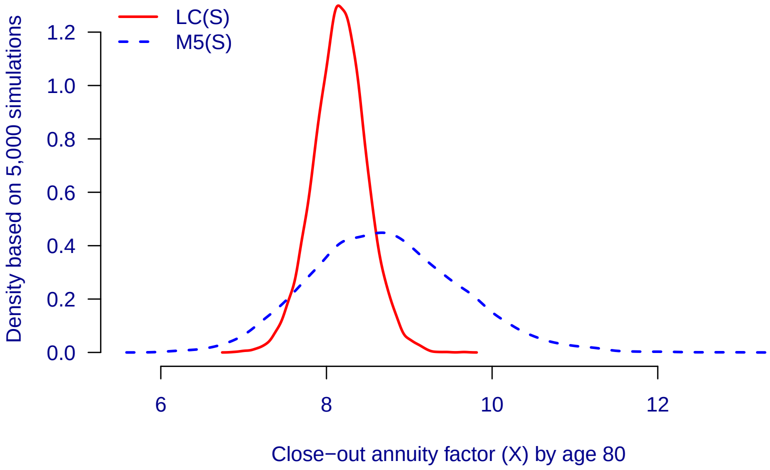
- How different can the answers get?
- Consider the spread at various ages under CBD model (M5)…

7 M5 sample paths — age 70



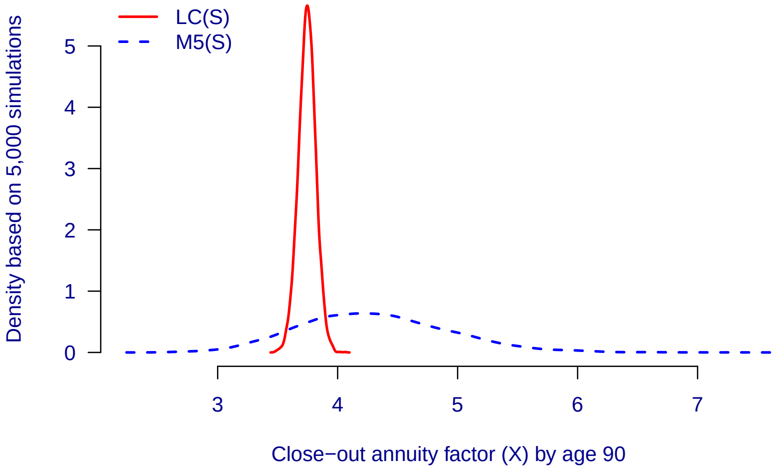
VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model. Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.

7 M5 sample paths — age 80



VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model. Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.

7 M5 sample paths — age 90



VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model. Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.

7 Index-based hedges

Variations to explore in future research:

- Different payoff functions.
- Valuing options to close out early.

If you are interested in the above, let me know!

8 Conclusions

8 Conclusions

- Longevity trend risk can be put into a one-year framework.
- Same outputs can be used for both VaR- and CTE-style solvency regimes.
- Framework extends to ORSA for insurers...

  ...and "glide paths" to buy-outs

  ...and assessing index-based hedges.
- Model risk is critical throughout.
- Expert judgement required for solvency capital...

  ...and valuation of index-based hedges.

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References IV

More on longevity risk at www.longevitas.co.uk